Relativistic Universe

Special Relativity 2

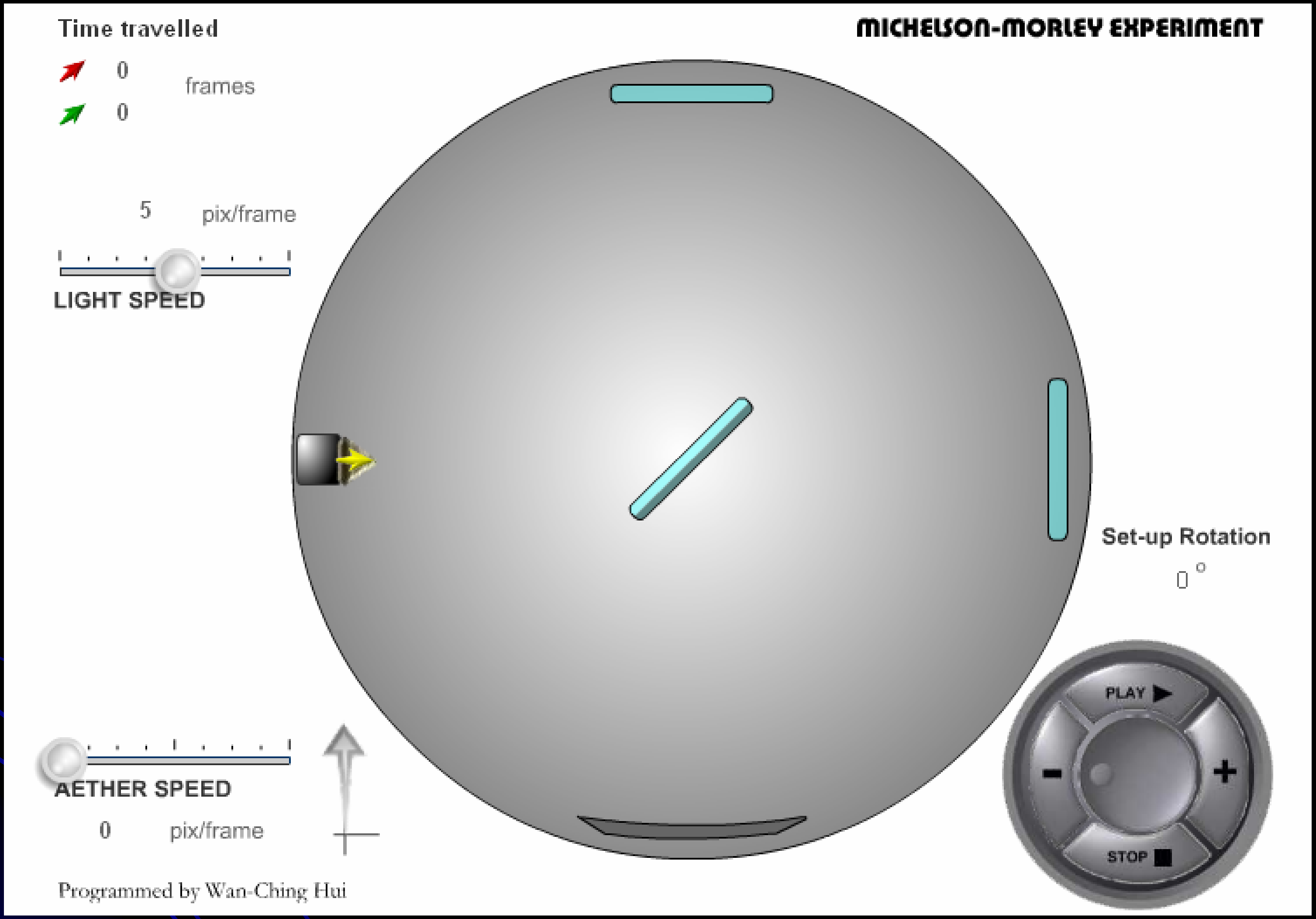
Postulates of SR

- Relativity Principle: physics is the same for all inertial observers.
- There exist a scale of velocity, which is observer-independent; this scale is identified with the velocity of light *c (e.g., velocity of propagation of photons and e-m waves.)*

But what do these postulate mean !?

The Michelson-Morley experiment

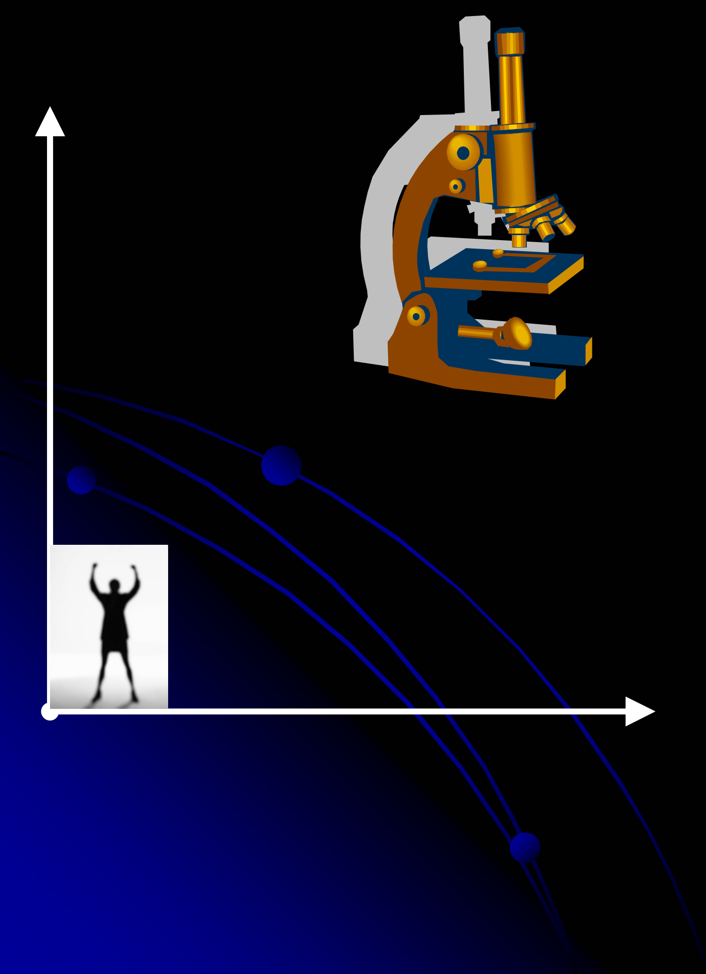
- Is there any medium carrying light, like air (say) carries sound waves? Assuming that such medium exists, and that Earth is moving with respect to it with velocity U, we should see an effect of order of U^2/c^2 in the following experiment.

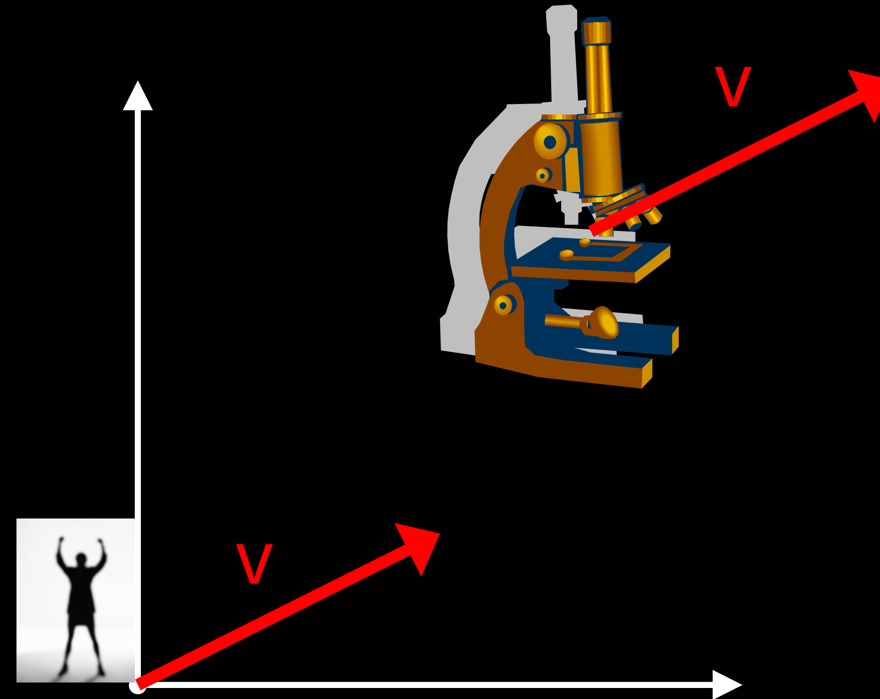

Time travelled
0
0
frames
5
pix/frame
LIGHT SPEED
AETHER SPEED
0
pix/frame
Programmed by Wan-Ching Hui
MICHELSON-MORLEY EXPERIMENT
Set-up Rotation
0 °
PLAY
STOP
-
+

- However there is no effect, which has led Einstein to abandoning ether altogether and postulating that the speed of light is an observer-independent quantity. Thus if two (inertial) observers measure speed of light, they got the same result. (But we must carefully define what does it mean ``to measure''.)

Relativity principle

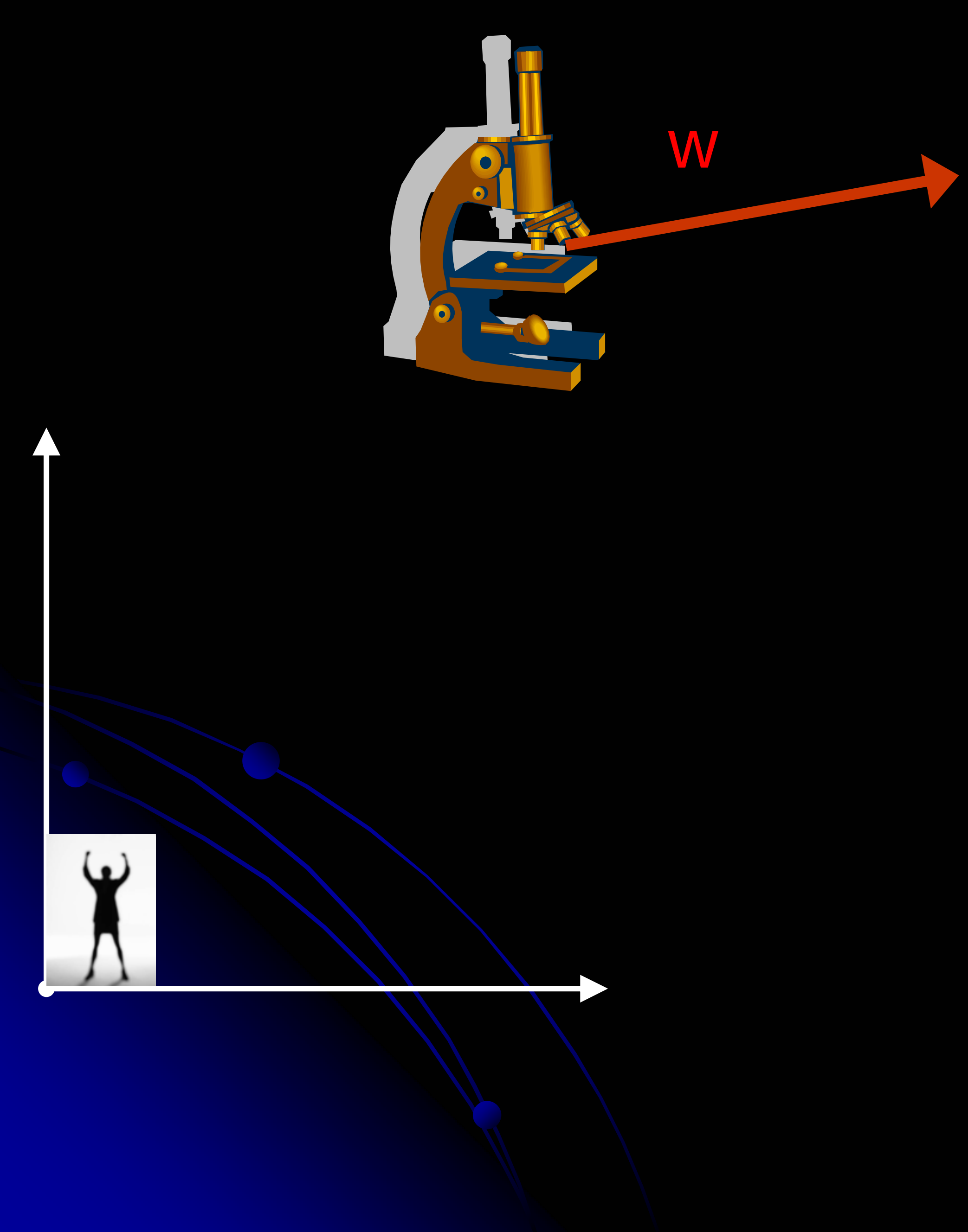
- First of all we must assume that one inertial system exists. Then all other systems (observers) moving uniformly with respect to this inertial system are inertial. Then relativity principle tells you that if two inertial observers make the same experiment, they get the same result.

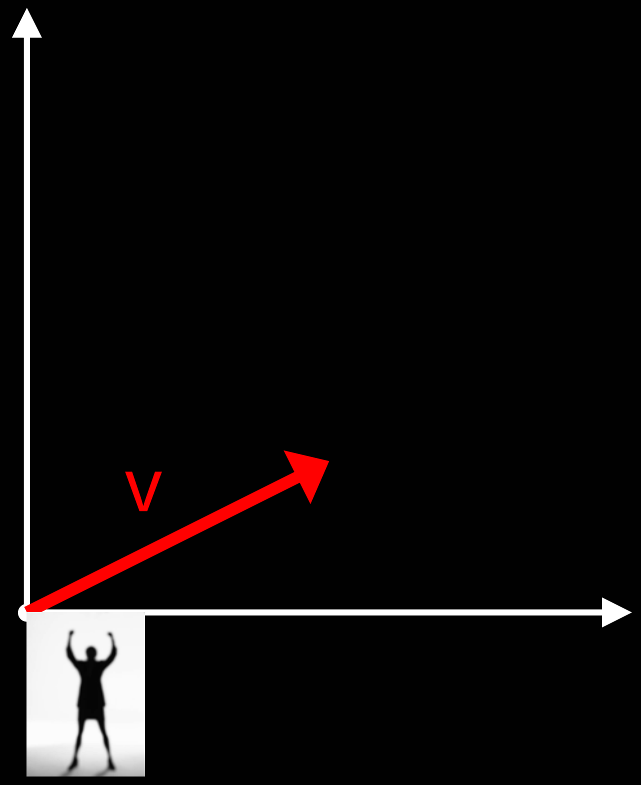




The results of both experiments are the same

- But this is not exactly what we want. We want to compare an observation of a single phenomena, as viewed from two different systems. So we are more interested in the following setting.





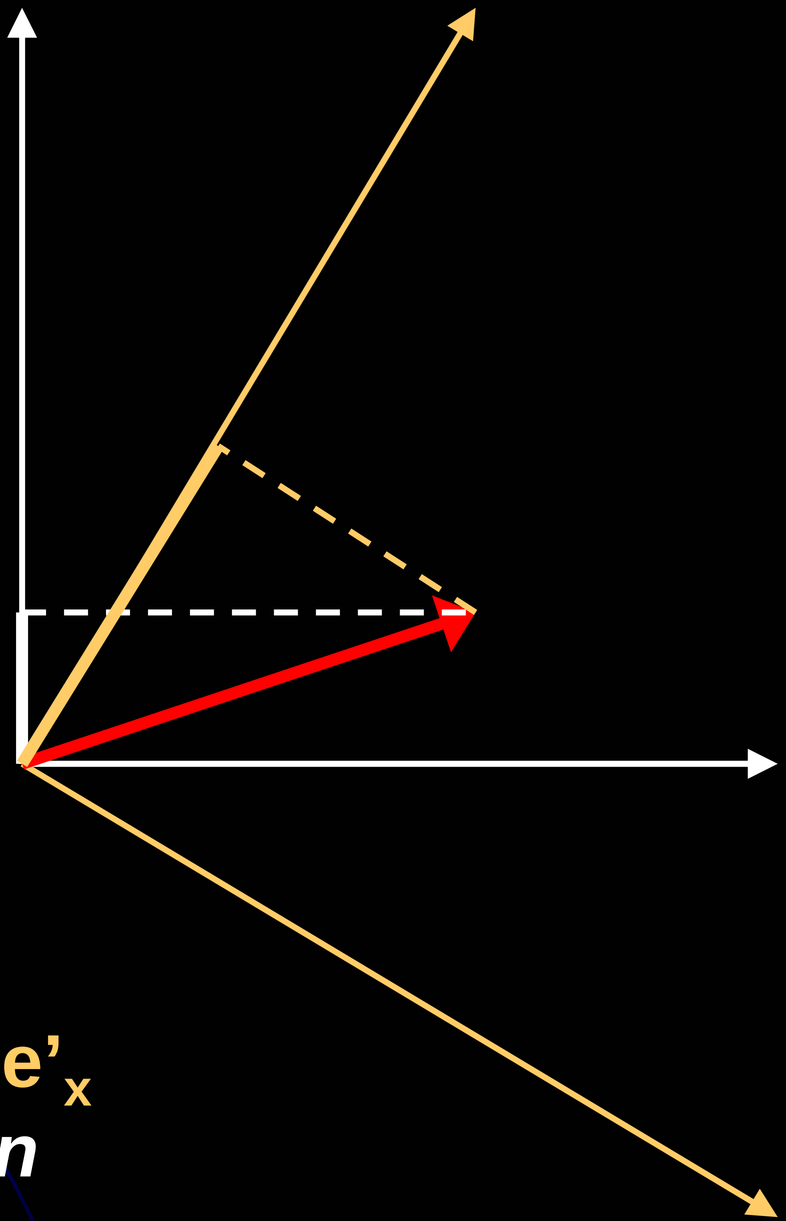
Now two observers compare *different* experiments!

Relativity principle

- In any inertial system both the *form* of physical laws and the *values of constants* in these laws are identical.
- This does not mean that the *results of measurements of physical quantities* are identical, but no measurement makes it possible to distinguish a particular inertial system.

- In fact the result of measurement is an *invariant*, and the relativity principle guarantees that we can relate these invariants to each other.

- To understand this consider the measurement of components of a vector in two systems.



First gives v e_x

Second gives v e'_x

Both are rotation invariant

- But since neither is distinguished, there must be a one-to-one relation between the results of measurements (given by the action of rotation group.)
- In SR we have to do with ``rotations’’ in space-time or in the space of energy and momenta.

Energy-momentum transformation

$$\boxed{E^2 - p^2c^2 = m^2c^4 = E'^2 - p'^2c^2}$$

$$\boxed{\begin{aligned} E' &= E\cosh\xi - cp\sinh\xi \\ p' &= p\cosh\xi - E/c\sinh\xi \end{aligned}}$$

- But remember: all quantities, *E, p, E', p'* ***are invariant!***

Space-time transformations

- Assume that the phase of a wave is invariant:

$$Et - px = E't' - p'x'$$

- Then it follows that:

$$t' = t\cosh\xi - \tfrac{x}{c}\sinh\xi$$
$$x' = x\cosh\xi - ct\sinh\xi$$

Rapidity and velocity

- But now we can relate rapidity ξ to velocity *v.* The point at rest in primed system moves with velocity *v* with respect to the unprimed one, so we have

$$\frac{x'}{t'} = 0 = \frac{x\cosh\xi - ct\sinh\xi}{t\cosh\xi - {}^{x}\!/_{c}\sinh\xi} = \frac{v\cosh\xi - c\sinh\xi}{\cosh\xi - {}^{v}\!/_{c}\sinh\xi}$$

$$v = c\tanh\xi$$

$$\cosh\xi = \frac{1}{\sqrt{1 - {}^{v^2}\!/_{c^2}}}, \quad \sinh\xi = \frac{v/c}{\sqrt{1 - {}^{v^2}\!/_{c^2}}}$$

Lorentz transformations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$c^2 t'^2 - x'^2 = c^2 t^2 - x^2$$

And now – back to physics

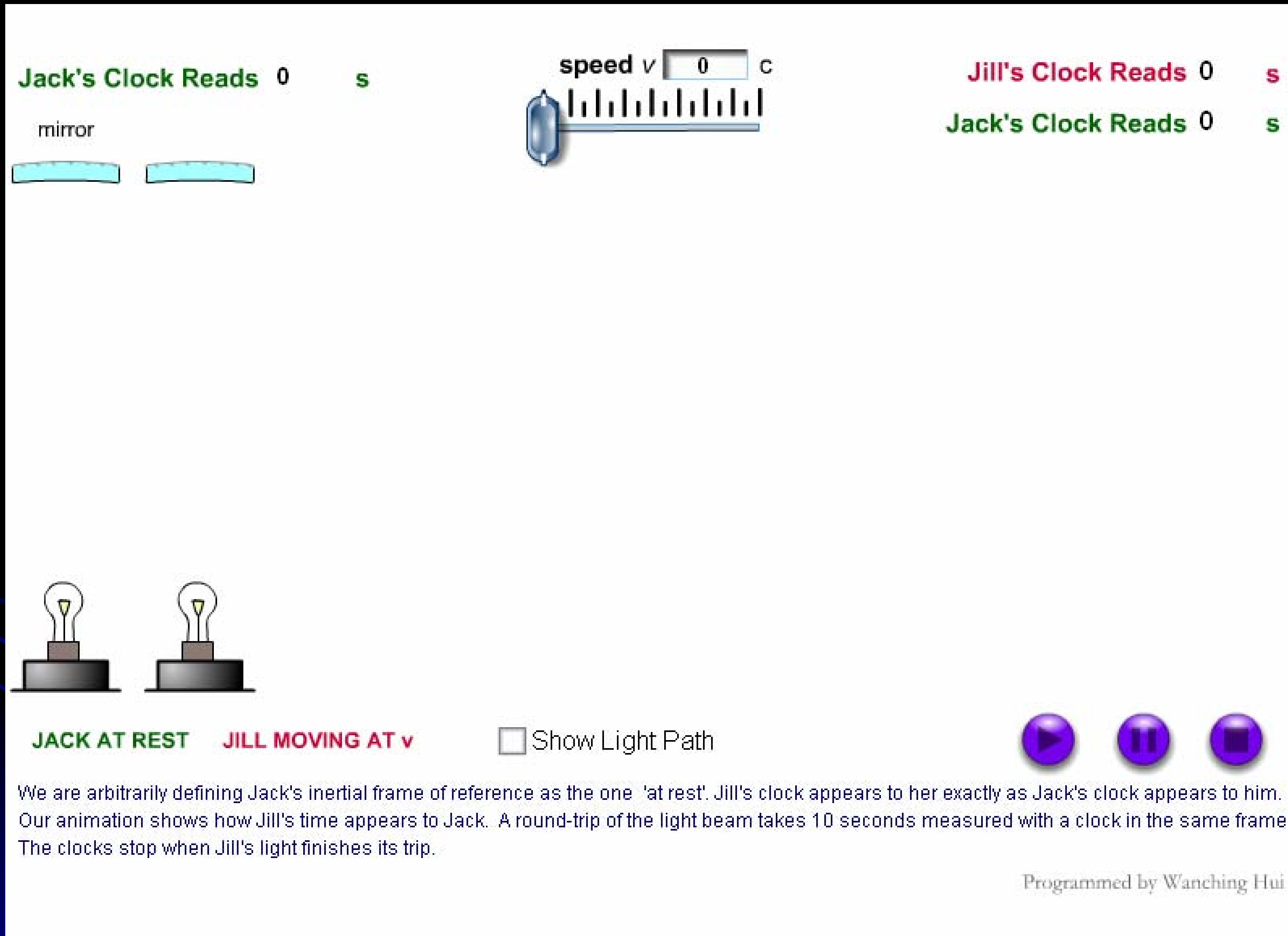
- All that I have done is just mathematics; we must check if it is consistent with natural physical definitions of space-time measurements.
- To make space-time measurements we need rulers and clocks.

Rulers

- By relativity principle the following recipe is just fine for any inertial observer: ``Take a given number of atoms, make them into cubic crystal; and use it as a ruler.''

Clocks

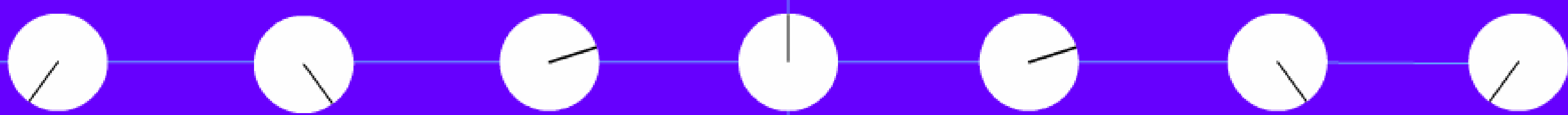
- In Galilean relativity (Newtonian physics) there exists an universal clock.
- In SR we can make a clock by using the ruler with mirror attached, placing it perpendicularly to the direction of motion and using light signals to measure time.


Jack's Clock Reads 0 s
mirror
speed v 0 c
Jill's Clock Reads 0 s
Jack's Clock Reads 0 s
JACK AT REST
JILL MOVING AT v
Show Light Path
We are arbitrarily defining Jack's inertial frame of reference as the one 'at rest'. Jill's clock appears to her exactly as Jack's clock appears to him. Our animation shows how Jill's time appears to Jack. A round-trip of the light beam takes 10 seconds measured with a clock in the same frame. The clocks stop when Jill's light finishes its trip.
Programmed by Wanching Hui

- But since there is no universal time we must synchronize clocks first at different positions, at rest with respect to each other, and then in different inertial frames.

The Synchronization of Clocks

We start with clocks placed at known measured distances from the origin in an inertial reference frame.



The clocks are not running yet.
They are waiting for a light signal from the
"Master Clock" at the origin of the inertial frame.

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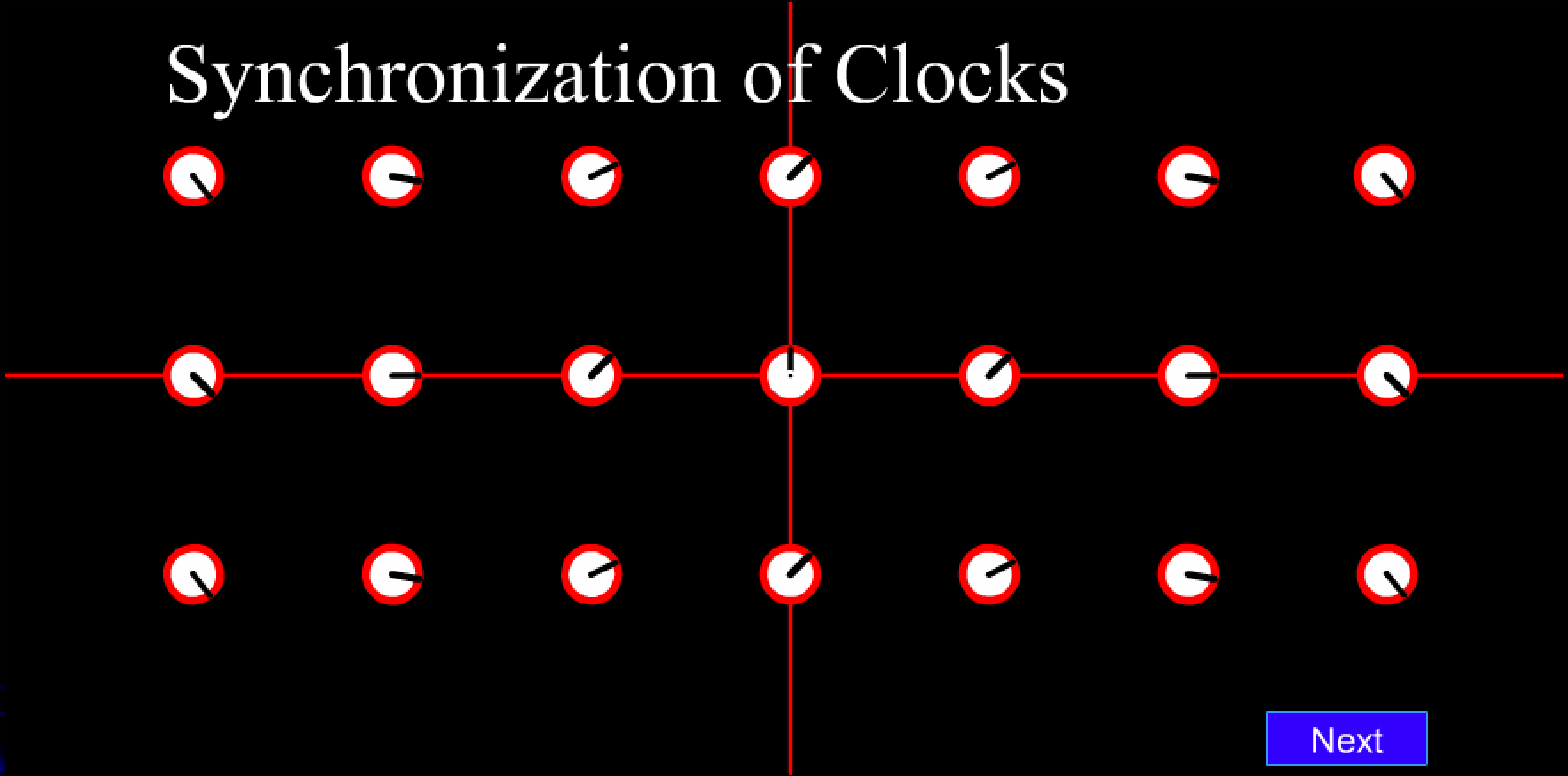
Synchronization of Clocks



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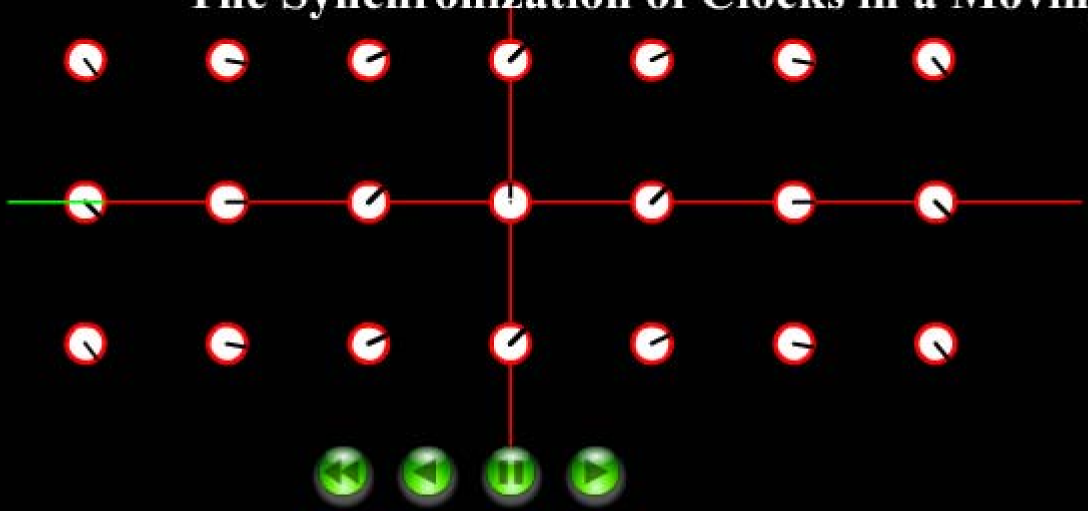
Clocks can be synchronized in a reference frame using a "master clock" at the origin and a light pulse to start the rest of the clocks.

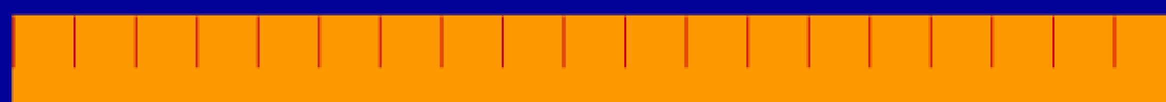
The Synchronization of Clocks in a Moving Reference System



Watch as the at rest reference system (red/unprimed) and the moving reference system (green/primed) undergo the synchronization process. Clocks in the (green/primed) are set ahead and stopped like those of the rest (red) reference system. Notice that the moving system will be lorentz contracted as viewed from the rest frame. The synchronizing light pulse for both frames will start when their origins coincide. Press the  button to start.

And now it's time for a show! Here are some relativistic effects

We can use a light clock moving with speed v to measure off a length on the ruler. Using arc burners we can burn the ruler at the begining and end of a "tick" of our moving clock..



*We call this length on a still ruler the "**Proper Length**"* ΔL_p

Next

A Light Clock at rest...

A light clock can be used to measure time. A "tick" of the clock is the time it takes the light to travel up to a mirror and back...

Next

Jack's Clock Reads 0 s

speed v 0 c

Jill's Clock Reads 0 s

Jack's Clock Reads 0 s

JACK AT REST JILL MOVING AT v

Show Light Path

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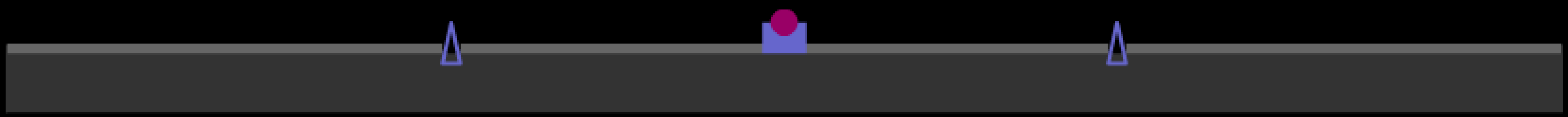
Programmed by Wanching Hui

A deep space station master awaits the nonstop passing of a starship traveling at 3/5th the speed of light.



He has placed two femto-second arc burners so as to burn the paint on the front and back of the starship as it passes. Why would he do such a thing???..... Just to prove a point!!!

He has the arc burners timed to fire at precisely the same time. He has placed a light detector midway between the arc burners. It will detect the flashes of light from the arc burners and will confirm their simultaneity....

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