The Pressure Soft Body
a Simple Model of Complex Behavior

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1. Abstract

SIMPLE computational model of soft body animation in two and three dimensions is introduced. The model is based on fundamental principles: Newton's Laws, Hookes Law and Ideal Gas Law. The simulated body consists of material points connected by springs. Additionally, pressure force on the body surface is applied. An equation of motion for each point is solved. The advantages of the model for educational applications are: it is based on fundamental physics laws; it is very fast and allows for an interactive simulation on home PC; its concept is simple enough to be understood by students. The model may be used as a representative example of physically based modelling in computer graphics and animation.

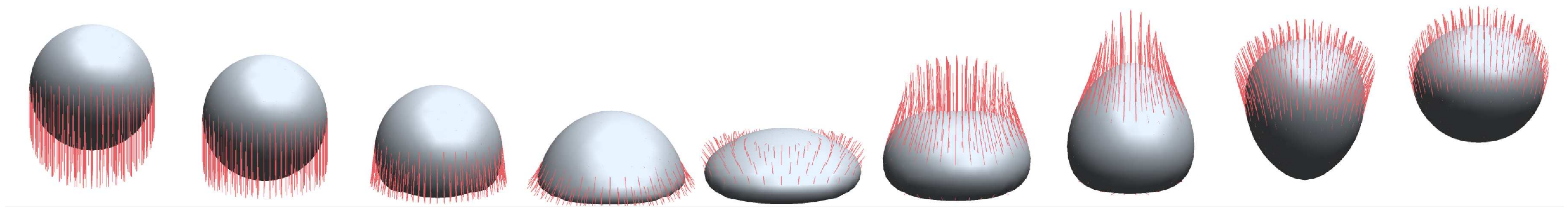

Figure 1: *The pressurized soft ball falling on the ground. The red lines indicate local velocity vectors of mass points building the shape of the object.*

2. Spring-Mass model

WE start from spring-mass model [1] that has been widely used in computer graphics and animation - mostly due to the simplicity and flexibility of its original form.

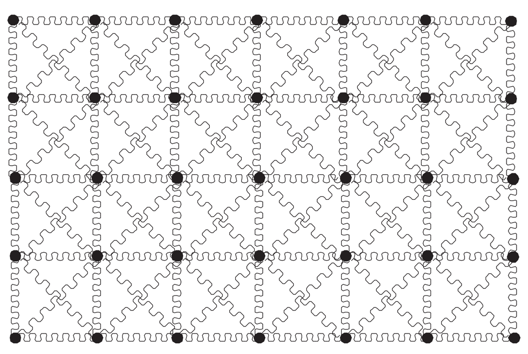

Figure 2: *The spring-mass model of cloth dynamics*

The spring-mass model contains mass particles connected by linear springs in both off- and diagonal directions (see figure 2). The Newton's laws of motion give ordinary differential equations describing dynamics of the model:

$$\frac{\mathrm{d}^2\vec{r}_i}{\mathrm{d}t^2} = \vec{F}_i/m_i,$$

where m_i is the i-th point mass, $\vec{r}_i$ is it position and $\vec{F}_i = \vec{F}^g_i + \vec{F}^w_i + \vec{F}^s_i + \ldots$ is a resultant force.
Starting with initial conditions for position $\vec{r}$ and velocity $\vec{v}$ above equation can be solved numerically with standard integration techniques (i.e. Runge-Kutta methods [4]).

3. Forces

Spring force between i-th and j-th are calculated using Hooke's law with damping term:

$$\vec{F}^s_{ij} = (|\vec{r}_i - \vec{r}_j| - r_0)\cdot k_s + (\vec{v}_i - \vec{v}_j)\cdot\left(\frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|}\right)\cdot k_d,$$

where:
- $\vec{v}_i$, is i-th point velocity vector,
- k_s and k_d are spring and damping constants.

Wind is approximated as a force acting on the point:

$$\vec{F}^w_i = ((\vec{w} - \vec{v}_i)\cdot\hat{n}_i)\cdot\hat{n}_i,$$

where $\vec{w}$ is the wind vector and $\hat{n}_i$ is normal-to-point vector (calculated from neighboring surfaces). External body forces (i.e. gravity $\vec{F}^g_i$) can be applied directly.

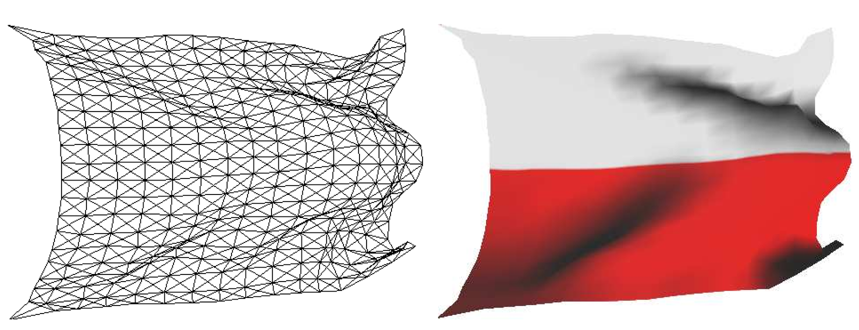

Figure 3: *An example of flag simulation. The flag is build of springs (left) and visualized with texture (right). Left-up and Left-bottom points are fixed. The wind vector* $\vec{w} = (1, 0, 0)$.

4. Pressurized Body Model (PSB)

We weave the object using the mass-spring model as shown in the figure 4.

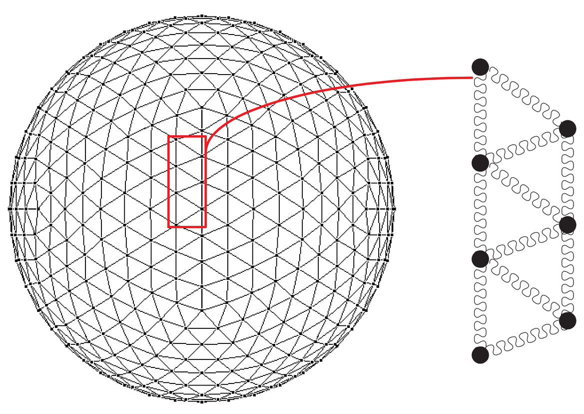

Figure 4: *Closed shapes woven with spring-mass model.*

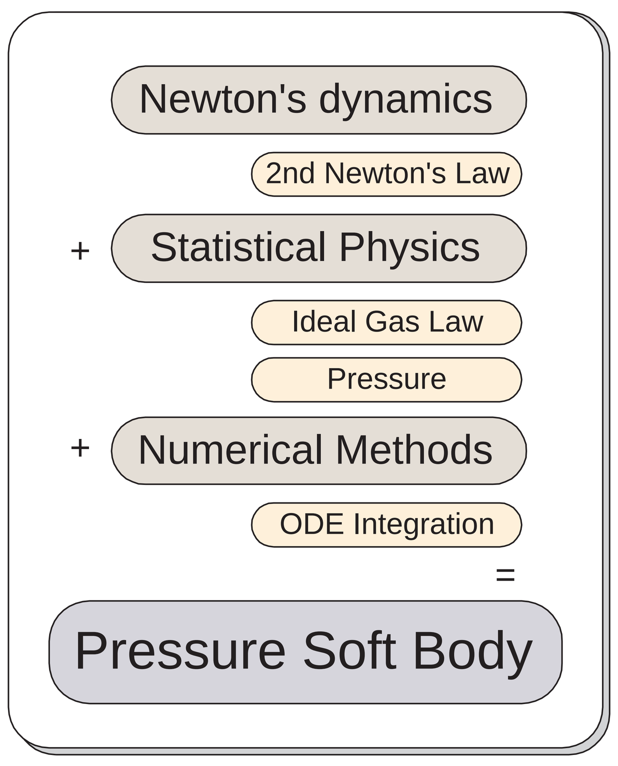

Figure 5: *Physics & methods behind the PSB model.*

We assume that the body is filled with a gas under pressure P higher than atmospheric pressure P_0 (see figure 6). Due to pressure difference an additional pressure force, that keeps the body shape, exists [3].

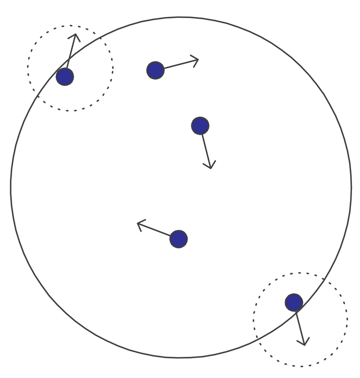

Figure 6: *The body is filled with a gas under high pressure.*

In order to approximate the pressure force, we assume that the body is large in comparison to gas particles and utilize the ideal gas law that balances pressure P and volume of the body V:

$$\mathrm{PV} = \mathrm{nRT}, \tag{1}$$

where R is the gas constant. An amount of the gas (n moles) and its temperature T is assumed to be constant too. Simple relation holds: $\mathrm{P} \propto 1/\mathrm{V}$ which shows that in order to calculate pressure, one have to calculate volume of the body that can be done using bounding objects or utilizing Gauss theorem (i.e. [2]).
The pressure force acting on i-th triangular surface can be calculated as follows:

$$\vec{F}^p_i = P\left(\hat{n}_i \cdot \mathrm{dS}_i\right), \tag{2}$$

where P is pressure calculated from the ideal gas law, $\hat{n}$ is normal to the face and dS is an area of the face.

5. Conclusion

We have shown a simple model of soft body dynamics. Because of its simplicity, it can be easy understood by students. On the other hand, it utilizes some fundamental physics, mathematics and numerical methods (see figure 5) that can be introduced and explained within it.

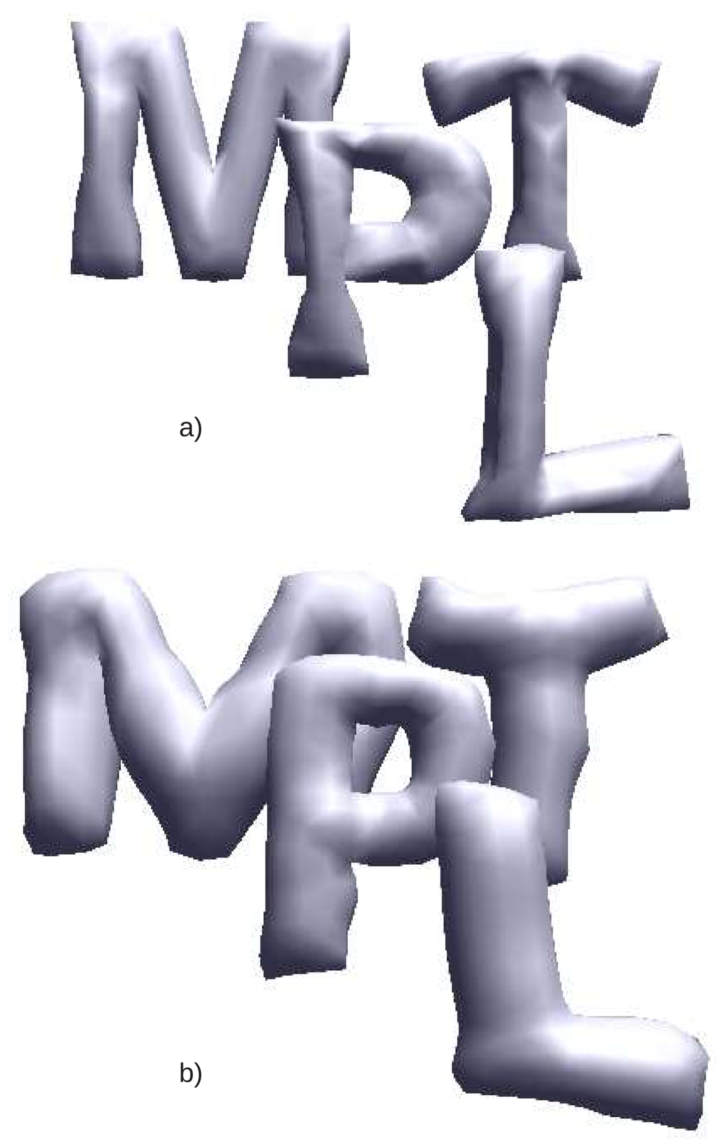

Figure 7: *The same objects under different lower (top) and higher (bottom) pressure.*

6. Acknowledgements

I would like to thank Przemysław Kuca who prepared 3d models of MPTL letters.

References

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