POLONEZ BIS

NATIONAL SCIENCE CENTRE
POLAND

Uniwersytet Wrocławski
Institute of Theoretical Physics

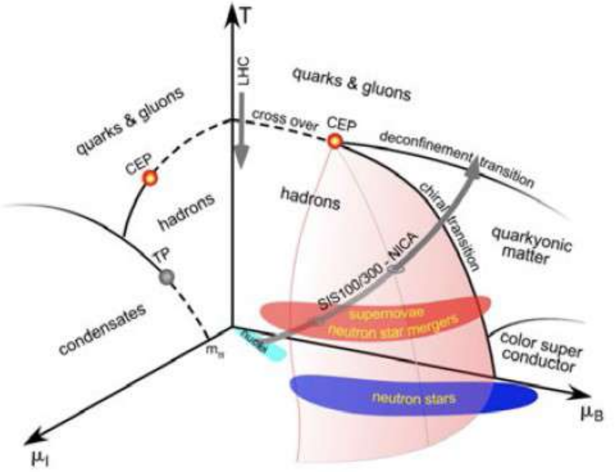
Bayesian analysis of the equation of state of dense nuclear matter

“BAYES“

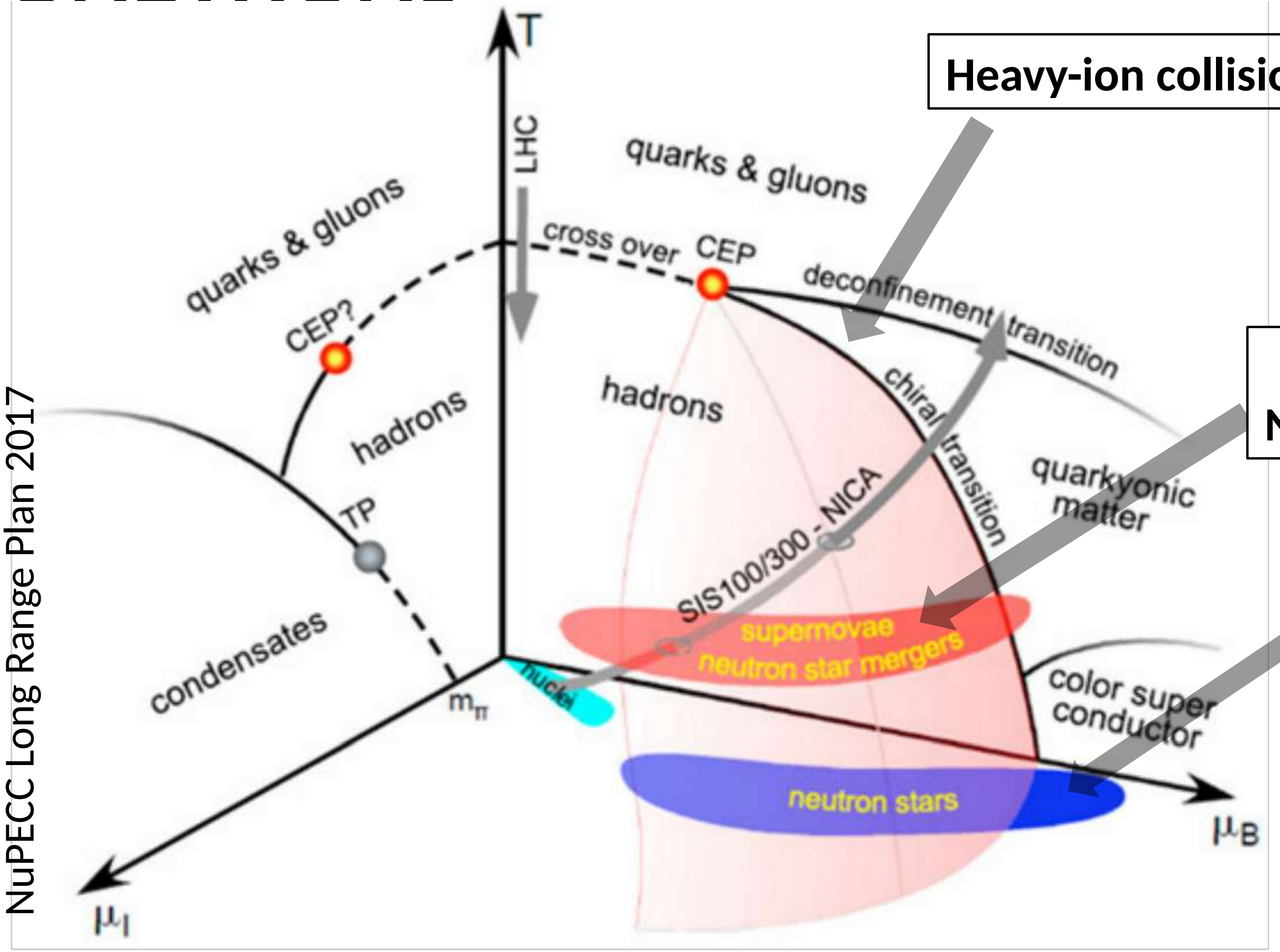
Project No. 2021/43/P/ST2/03319

ALEXANDER AYRIYAN

JANUARY, 10TH, 2025



NUCLEAR MATTER UNDER EXTREME CONDITIONS



NuPECC Long Range Plan 2017

QCD phase diagram

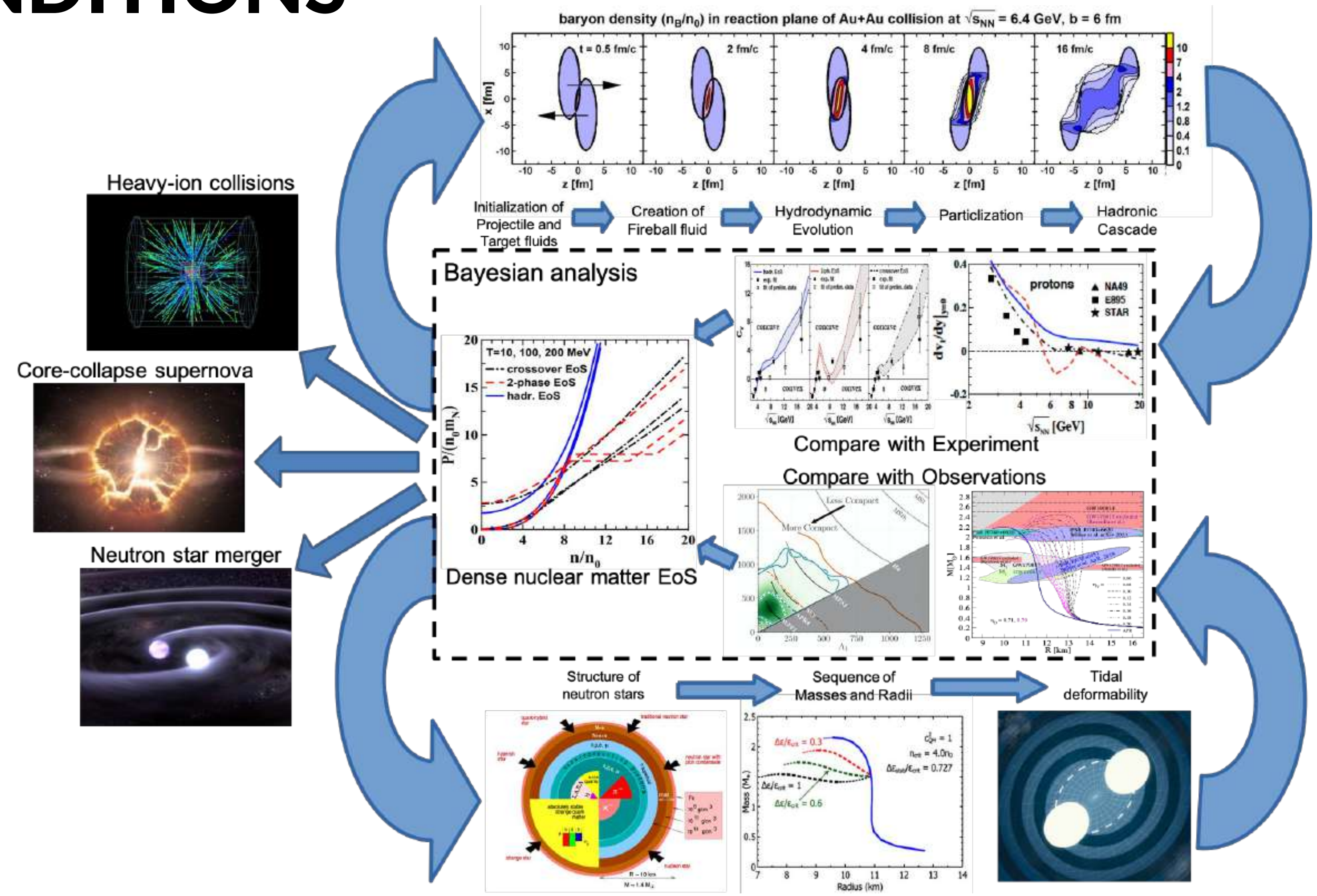
Heavy-ion collisions (HIC)

Supernova and Neutron star merger

Neutron stars

Now, both Stellar and HIC matter will be studied with a unified EoS

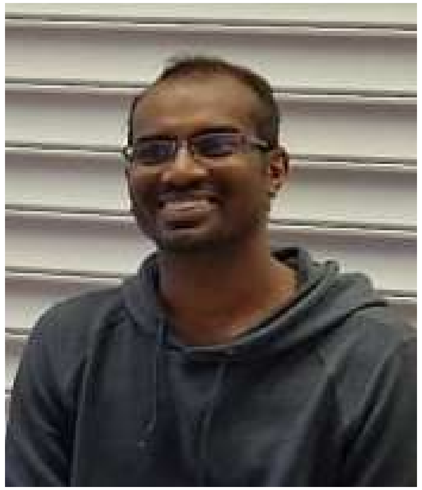
NUCLEAR MATTER UNDER EXTREME CONDITIONS



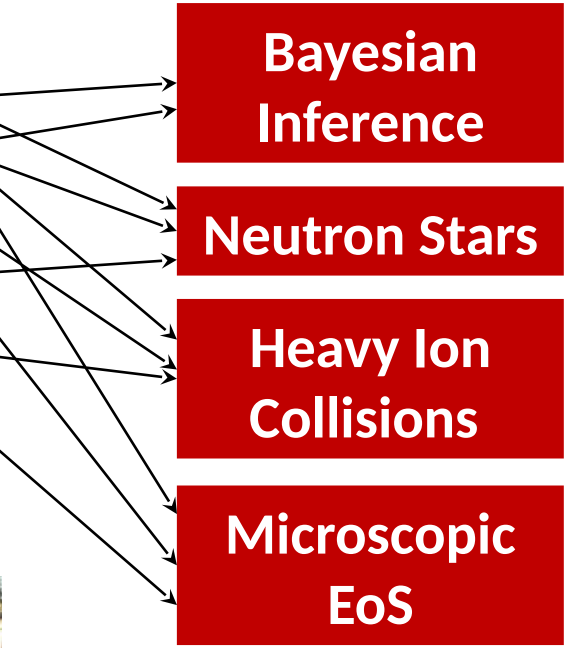
QCD phase diagram

PROJECT GROUP

Elizaveta



Biplab



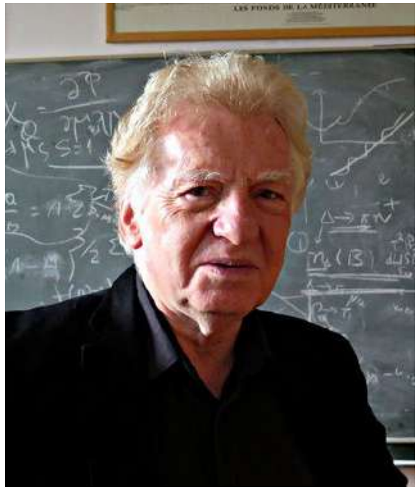
Alexander Ayriyan – PI
David Blaschke – Mentor
Ludwik Turko – Co-investigator
Oleksii Ivanytskyi – Co-investigator
Elizaveta Zherebtsova – PhD student
Biplab Mahato – PhD student

Bayesian Inference

Neutron Stars

Heavy Ion Collisions

Microscopic EoS



Ludwik

Oleksii

David

Alexander

UNIFIED EOS

- Medium dependent excluded-volume mechanism for RDF (DD2)

- Nuclear RMF (Walecka) model + chiral quark (nonlocal PNJL) model with switching function or two-zone interpolation

Typel & Blaschke. Universe 4, 32 (2018)
Typel. *Universal equation of state with modified excluded-volume mechanism* [in progress]
Ivanytskyi & Blaschke. Eur. Phys. J. A 58(8) 152 (2022)
Albright, Kapusta & Young. PRC90, 024915 (2014); Kapusta & Welle. PRC106, 044901 (2022)

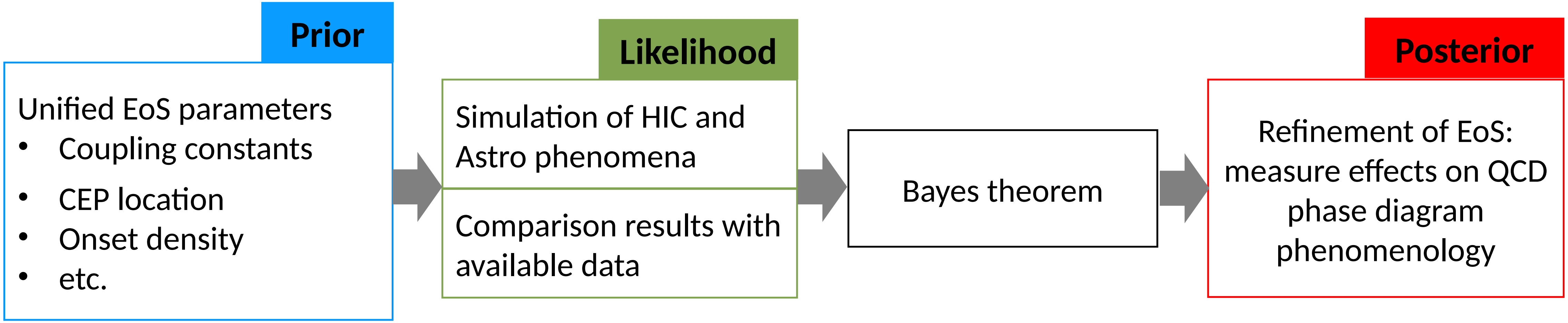
WHY BAYESIAN INFERENCE?

Bayes' theorem:

$$\underbrace{p(H_1 \mid D, I)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H_1, I)}^{\text{Likelihood}}\;\overbrace{p(H_1 \mid I)}^{\text{Prior}}}{\underbrace{p(D \mid I)}_{\text{Evidence}}}$$

Hypothesis (H1 = EoS parameter set 1)

Prior: knowledge before experiment (logically)
Likelihood: Probability for data if the hypothesis was true
Posterior: Probability that the hypothesis is true given the data
Evidence: normalization; important for model comparison



Prior

Unified EoS parameters

- Coupling constants
- CEP location
- Onset density
- etc.

Likelihood

Simulation of HIC and Astro phenomena

Comparison results with available data

Bayes theorem

Posterior

Refinement of EoS: measure effects on QCD phase diagram phenomenology

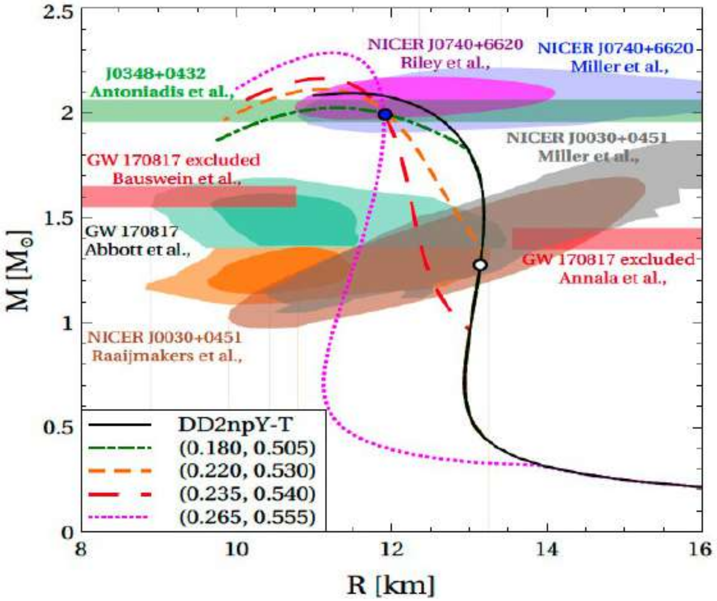
Our BA hypotheses concern the fundamental parameters of low energy QCD!

AVAILABLE HIC AND ASTROPHYSICAL DATA

Astrophysical data

- Lower limit of maximum mass
- Mass-radius measurement
- Tidal deformability from GW

etc.

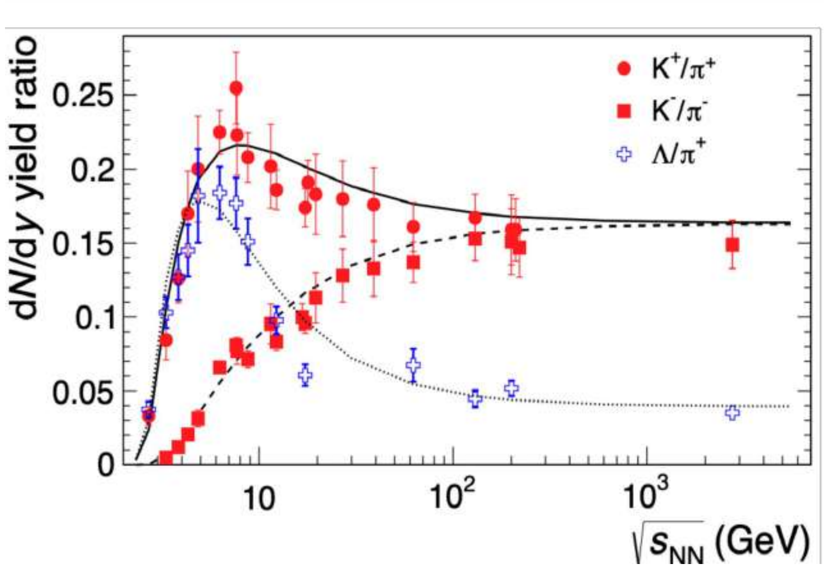


NICER, LIGO/VIRGO, SKA, etc.

HIC data

- Curvature at midrapidity
- Multipolar flow
- Hadron yield ratios (e.g., $\mathbf{K^+/\pi^+}$)

etc.



NA61@SPS, BES@RHIC, CBM@FAIR

HOST INSTITUTION

Uniwersytet Wrocławski

Institute of Theoretical Physics

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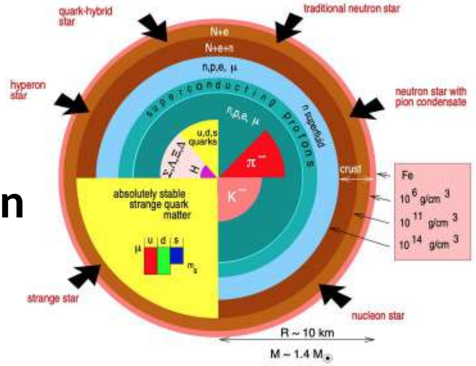
Astrophysics of Compact stars

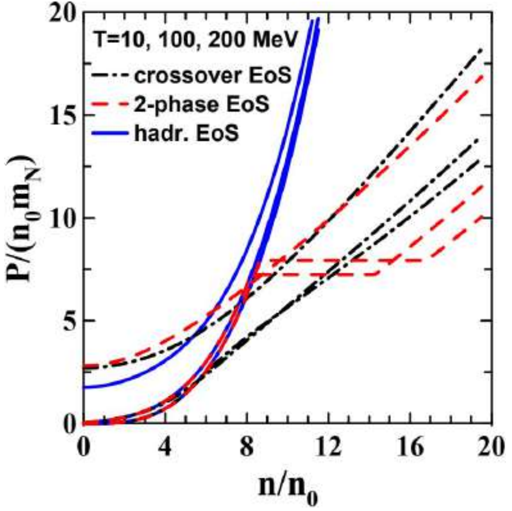


prof. D. Blaschke
prof. A. Sedrakian

Simulations of supernova explosions

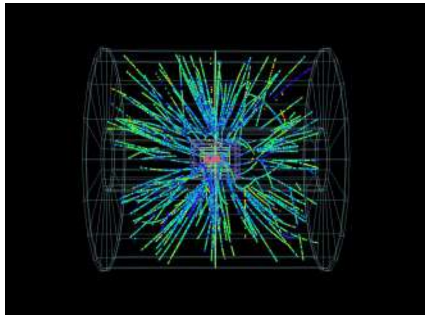
prof. T. Fischer

Theory of matter under extreme conditions



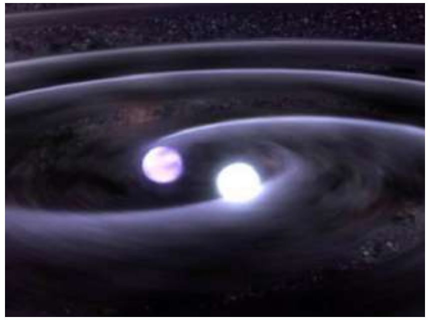
prof. D. Blaschke,
prof. K. Redlich,
prof. C. Sasaki
dr O. Ivanytskyi
dr S. Typel (GSI)

HIC and its simulation



prof. K. Redlich (ALICE)
prof. D. Blaschke
Prof. L. Turko (NA61)
dr P. Huovinen
dr Iu. Karpenko

Neutron star merger



prof. D. Blaschke
prof. A. Sedrakian
dr A. Bauswein (GSI)

EXPECTED RESULTS

For the first time a joint investigation with HIC and Astrophysical phenomena will contribute to

- Answering the open questions (CEP, CSC, onset density etc.)
- Providing general purpose EoS tables (to CompOSE data base)
- Contribution to White Paper “Hadron Physics at GSI and FAIR”

Studies of the color superconducting phase in the EoS will have impact on:

- Structure of the QCD phase diagram
- Simulations of low-energy HIC experiments
- Onset of deconfinement in NS and their mergers

Publications
PRC, PRD, Universe, EPJA etc.

Talks at conferences
CSQCD, CPOD, QM, SQM etc.

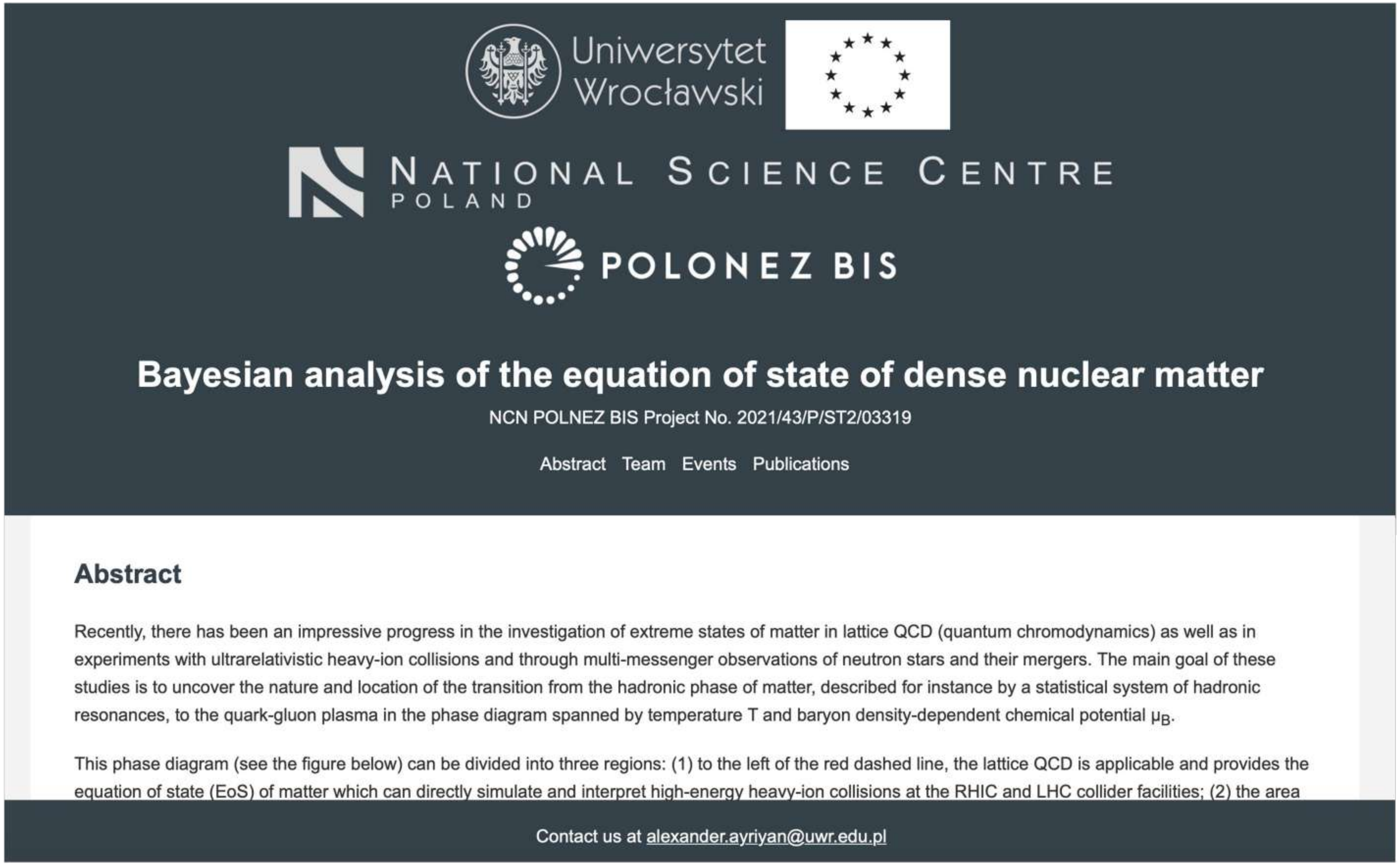
Exchange of knowledge

HI → PI

- Theory of matter under extreme conditions
- HIC phenomenology
- Boost international collaboration

PI → HI

- Joint investigation of EoS in CS and HIC
- Phase transition construction
- Computational and statistical methods BI, NN etc.



Bayesian analysis of the equation of state of dense nuclear matter

NCN POLNEZ BIS Project No. 2021/43/P/ST2/03319

Abstract

Recently, there has been an impressive progress in the investigation of extreme states of matter in lattice QCD (quantum chromodynamics) as well as in experiments with ultrarelativistic heavy-ion collisions and through multi-messenger observations of neutron stars and their mergers. The main goal of these studies is to uncover the nature and location of the transition from the hadronic phase of matter, described for instance by a statistical system of hadronic resonances, to the quark-gluon plasma in the phase diagram spanned by temperature T and baryon density-dependent chemical potential μ_B.

This phase diagram (see the figure below) can be divided into three regions: (1) to the left of the red dashed line, the lattice QCD is applicable and provides the equation of state (EoS) of matter which can directly simulate and interpret high-energy heavy-ion collisions at the RHIC and LHC collider facilities; (2) the area

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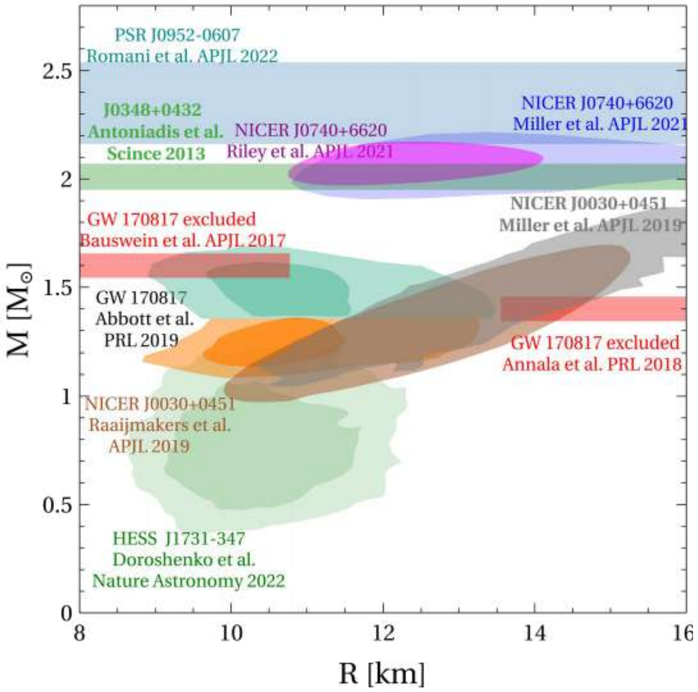
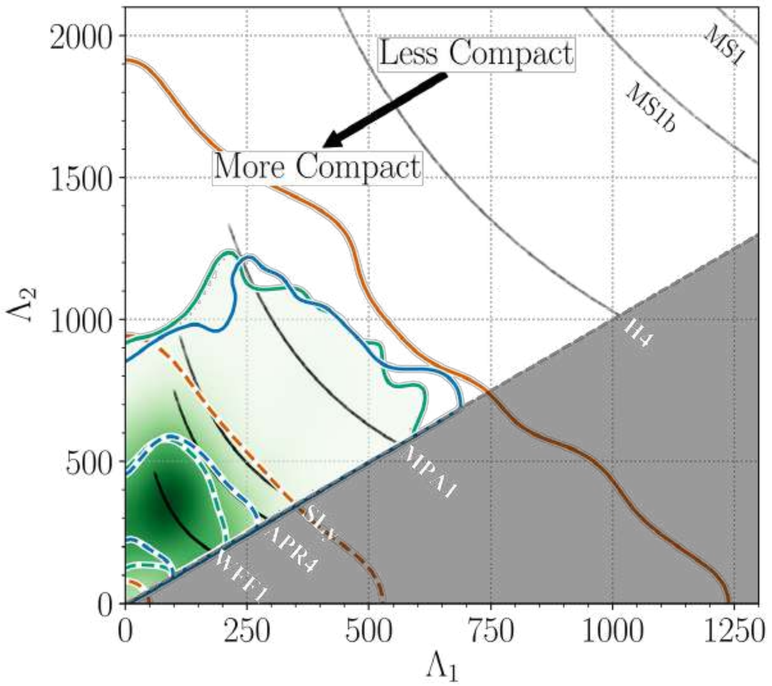
BAYESIAN INFERENCE FROM MULTI-MESSENGER ASTRONOMY

Agnostic
- Model-independent
- No evidence to physical quantities

Physics Informed
- Model-dependent
- Quantitative measure of physical parameters

Metamodeling
- Quasi-independent
- Quantitative measure of physical parameters





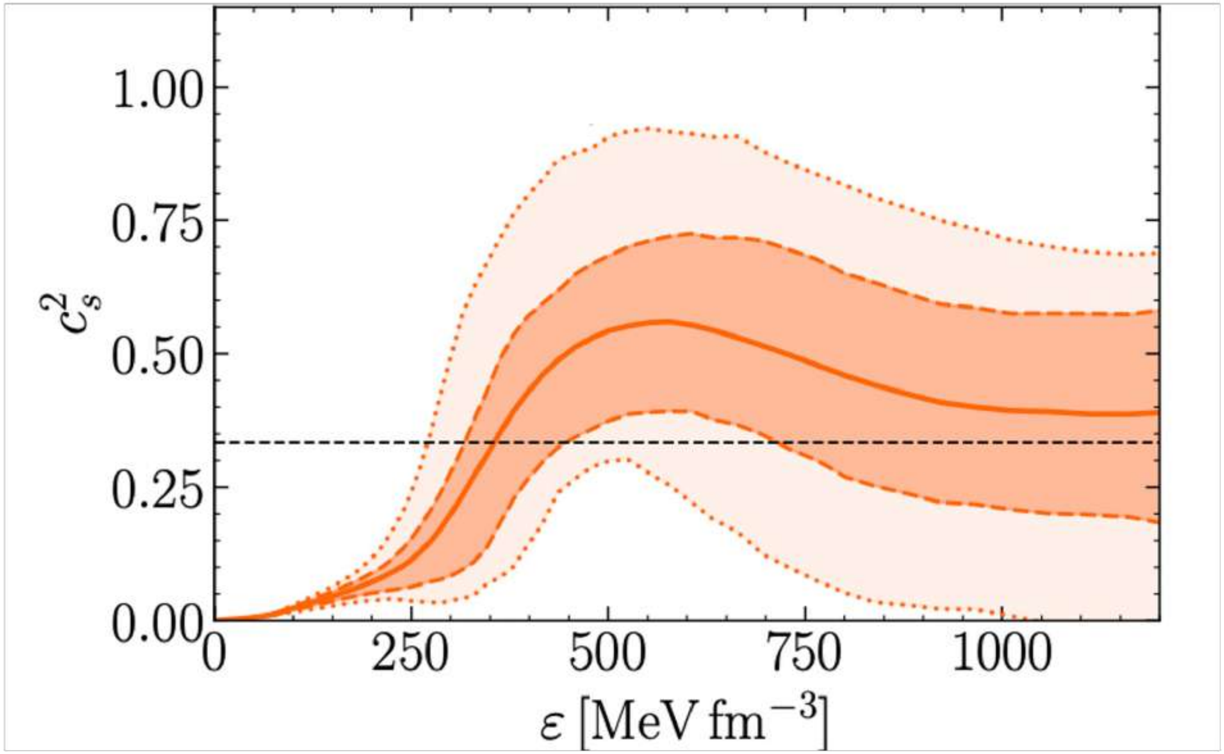
$$P(E_{MR}|\pi_i) = \int_{l_2} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau + \\ + \int_{l_3} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau$$

$$P(E_{GW}|\pi_i) = \int_{l_{22}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \\ + \int_{l_{32}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \\ + \int_{l_{23}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau \\ + \int_{l_{33}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau$$

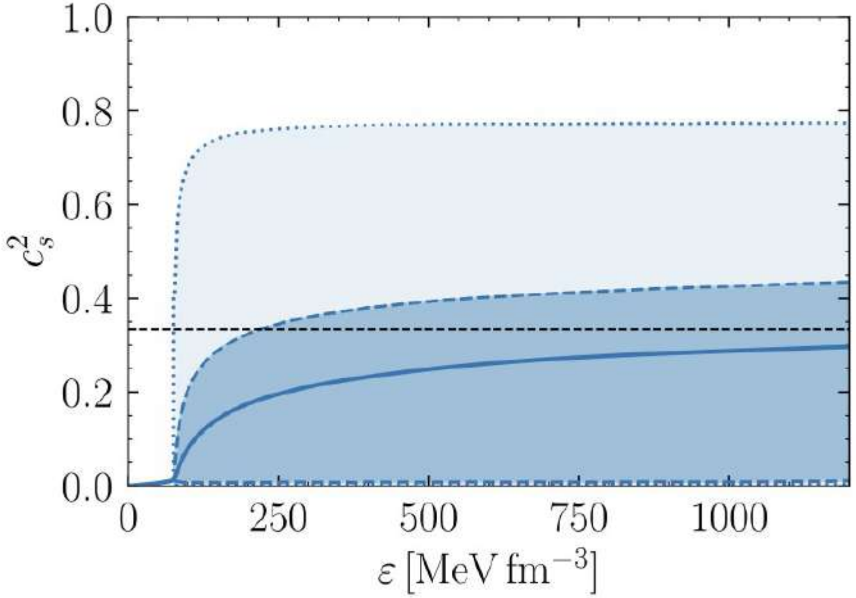
$$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{M_i - \mu_A}{\sqrt{2}\sigma_A}\right)\right]$$

AGNOSTIC BA

Brandes, Weise & Kaiser. PRD108, 094014 (2023)

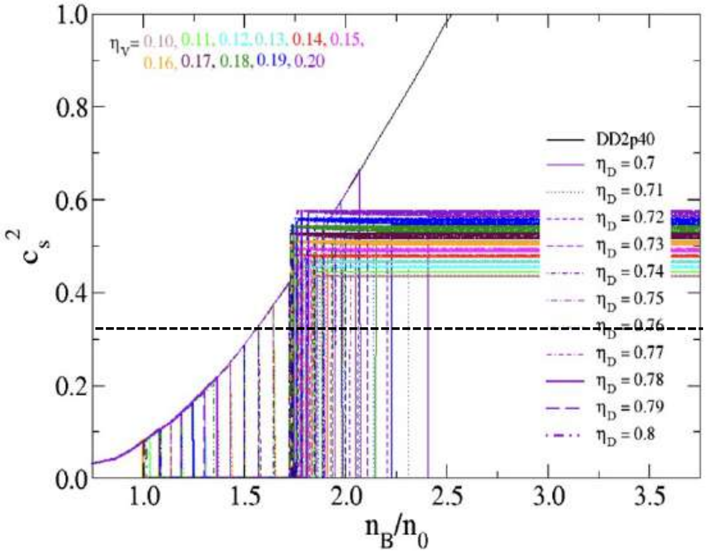


$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}$$

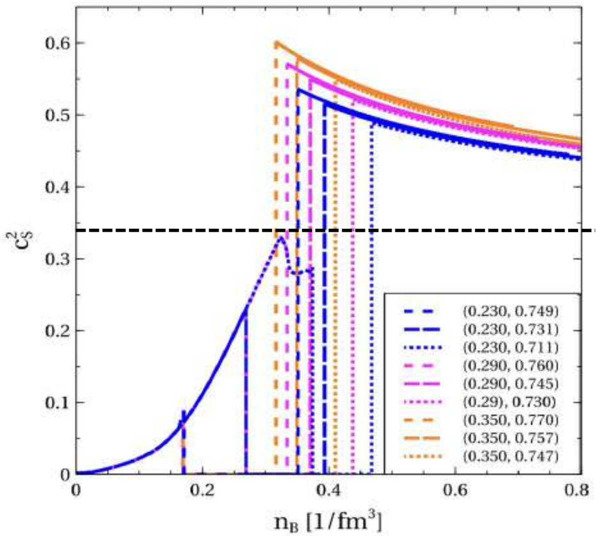


Annala et al. Nature Phys. 16, 907 (2020)
QM: c_s^2 <= 1/3

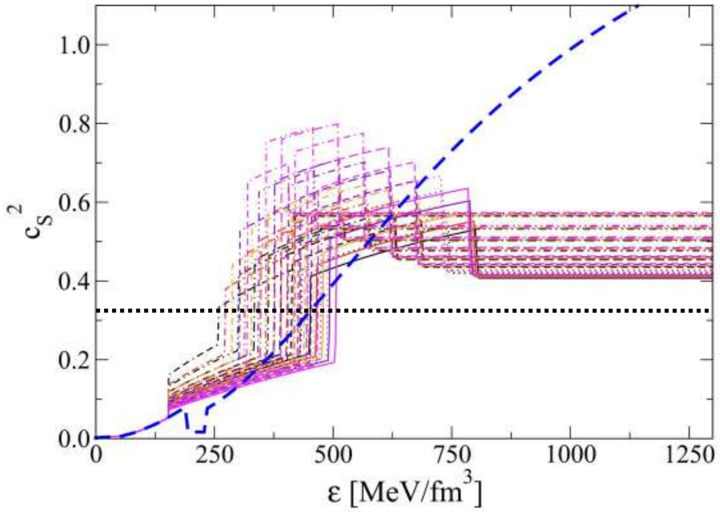
POSSIBLE PHASE TRANSTITION TO CSC MATTER



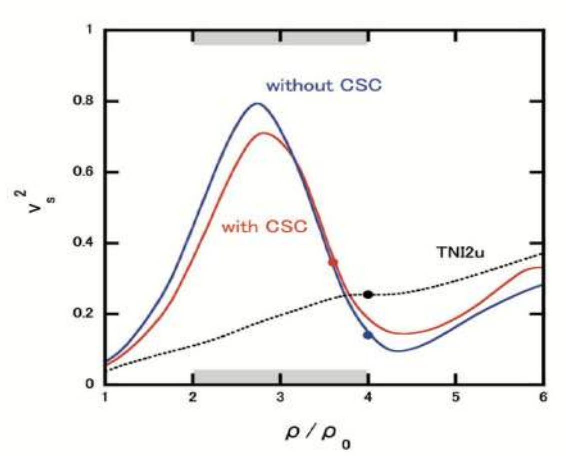
Shahrbaf et al. PRD107, 054011 (2023)



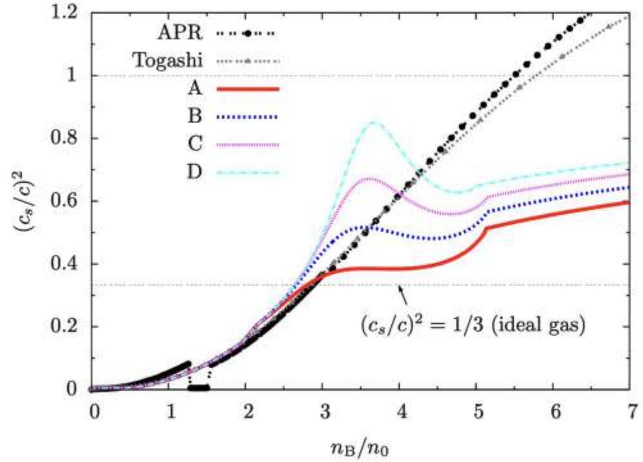
Gärtlein et al. PRD108, 114028 (2023)



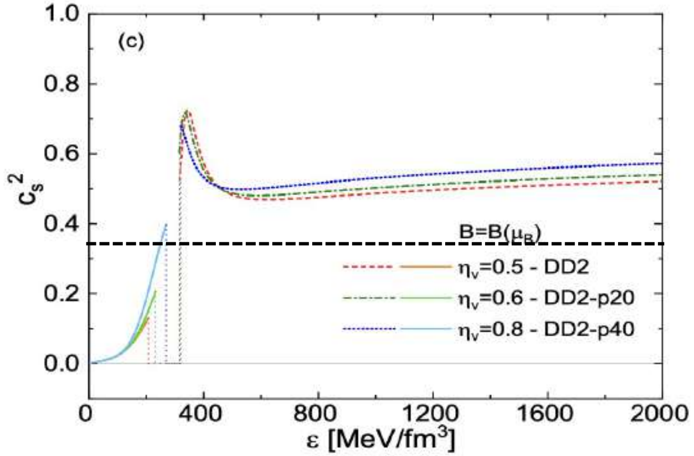
Ayriyan et al. EPJA57, 318 (2021)



Masuda, Hatsuda & Takatsuka, EPJA52, 65 (2016)



Baym et al., ApJ885, 42 (2019)



Contrera et al. PRC105, 045808 (2022)

DD2+NLNJL

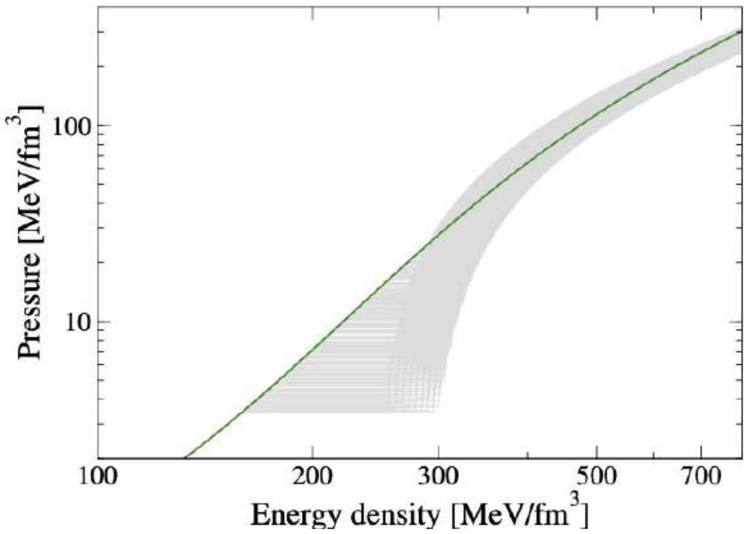
Two-flavor color-superconducting quark matter with 3DFF nonlocal chiral quark model defined by an effective Euclidean action functional that in the case of two light flavors is given by

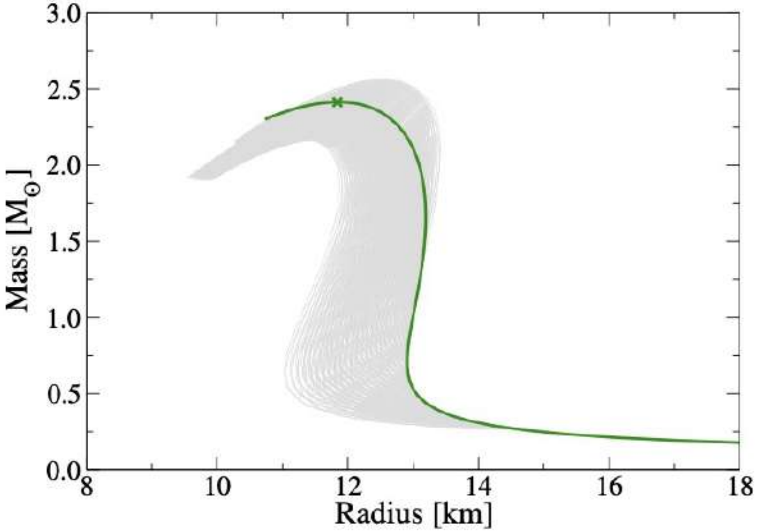
$$
\begin{aligned}
S_E \;&=\; \int d^4x \left\{ \bar\psi(x)\left(-i\partial\!\!\!/ + \hat m - \gamma_0\hat\mu\right)\psi(x) - \frac{G_S}{2}\left[j_S^f(x) j_S^f(x) \right.\right. \\
&+\; \left.\left. \eta_D \left[j_D^a(x)\right]^\dagger j_D^a(x) - \eta_V j_V^\mu(x)\, j_V^\mu(x) \right] \right\}.
\end{aligned}
$$

$$
\begin{aligned}
j_S^f(x) \;&=\; \int d^4z\, g_S(z)\, \bar\psi(x+\frac{z}{2})\,\Gamma_f\,\psi(x-\frac{z}{2})\,, \\
j_D^a(x) \;&=\; \int d^4z\, g_D(z)\, \bar\psi_C(x+\frac{z}{2})\, i\gamma_5\tau_2\lambda_a\,\psi(x-\frac{z}{2})\,, \\
j_V^\mu(x) \;&=\; \int d^4z\, g_V(z)\, \bar\psi(x+\frac{z}{2})\, i\gamma^\mu\,\psi(x-\frac{z}{2})\,,
\end{aligned}
$$

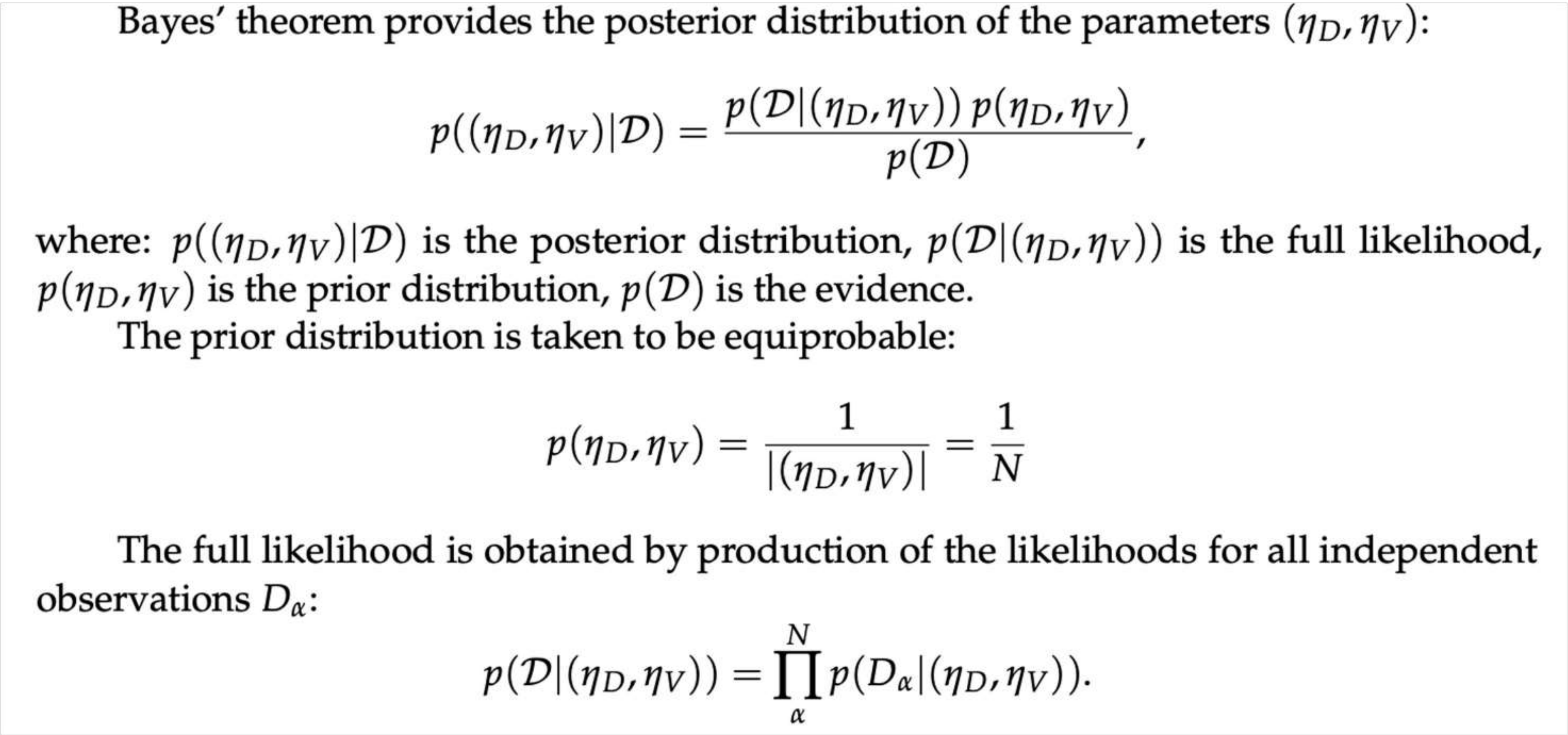
For the nonlocality a Gaussian ansatz is employed

$$g_i(\vec p) = \exp(-\vec p^2/\Lambda_i^2),\;\; i = S, D,$$





BA FORMALISM



Bayes' theorem provides the posterior distribution of the parameters (η_D, η_V):

$$p((\eta_D, \eta_V)|\mathcal{D}) = \frac{p(\mathcal{D}|(\eta_D, \eta_V))\, p(\eta_D, \eta_V)}{p(\mathcal{D})},$$

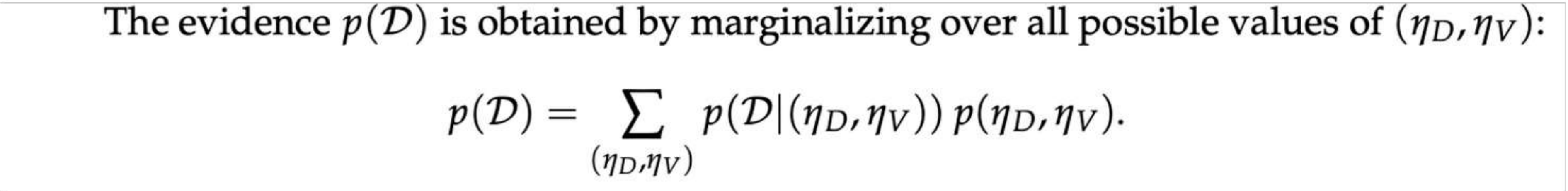
where: $p((\eta_D, \eta_V)|\mathcal{D})$ is the posterior distribution, $p(\mathcal{D}|(\eta_D, \eta_V))$ is the full likelihood, $p(\eta_D, \eta_V)$ is the prior distribution, $p(\mathcal{D})$ is the evidence.

The prior distribution is taken to be equiprobable:

$$p(\eta_D, \eta_V) = \frac{1}{|(\eta_D, \eta_V)|} = \frac{1}{N}$$

The full likelihood is obtained by production of the likelihoods for all independent observations D_α:

$$p(\mathcal{D}|(\eta_D, \eta_V)) = \prod_{\alpha}^{N} p(D_\alpha|(\eta_D, \eta_V)).$$

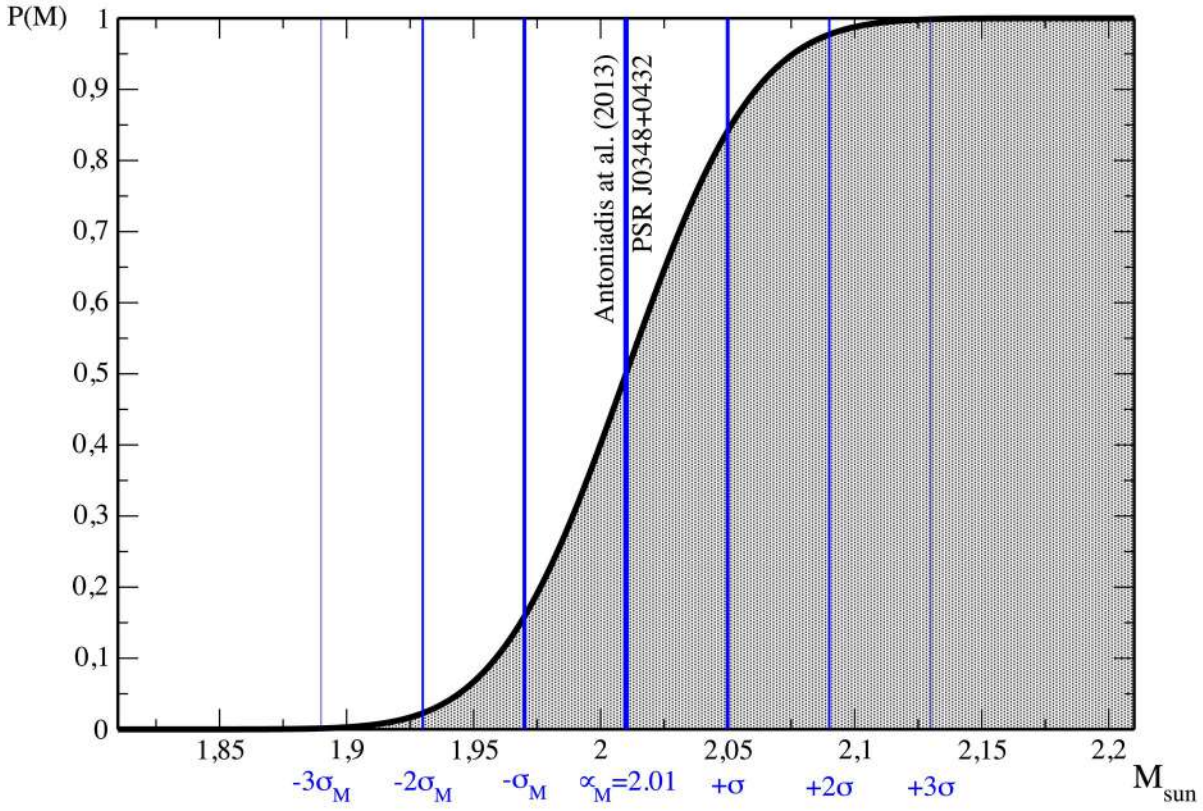


The evidence $p(\mathcal{D})$ is obtained by marginalizing over all possible values of (η_D, η_V):

$$p(\mathcal{D}) = \sum_{(\eta_D, \eta_V)} p(\mathcal{D}|(\eta_D, \eta_V))\, p(\eta_D, \eta_V).$$

LOWER LIMIT OF MAXIMUM MASS

$$p(D_{M_{\max}^{(i)}}|(\eta_D,\eta_V)) = F_{\mathcal{N}}(M_{\max}(\eta_D,\eta_V);\mu_M^{(i)},\sigma_M^{(i)})$$

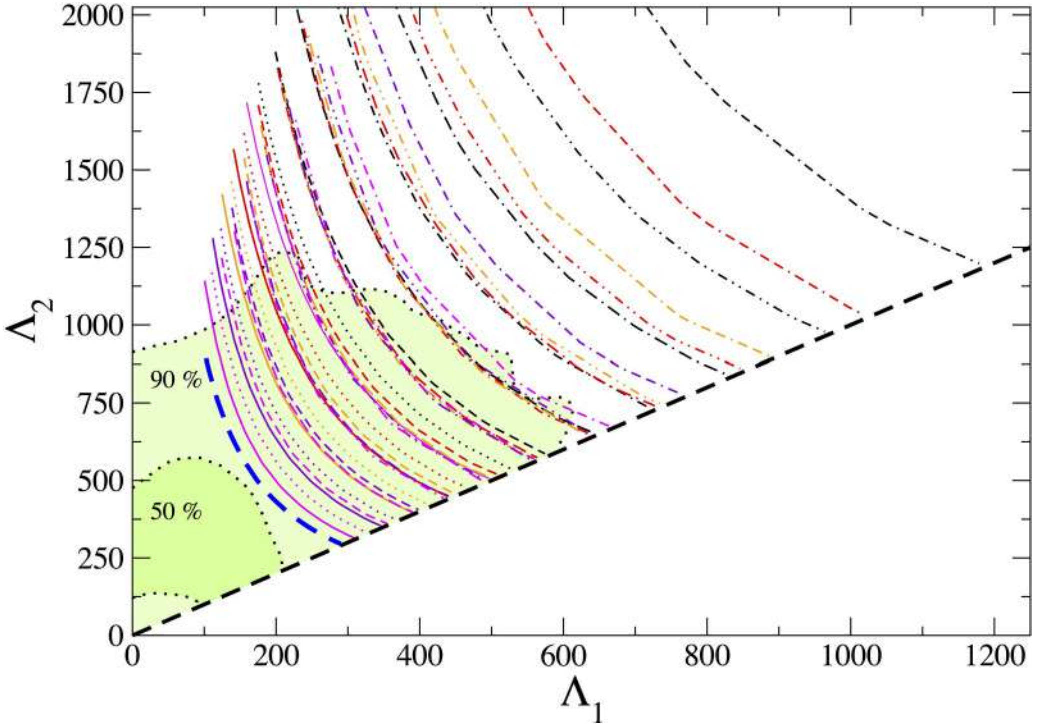


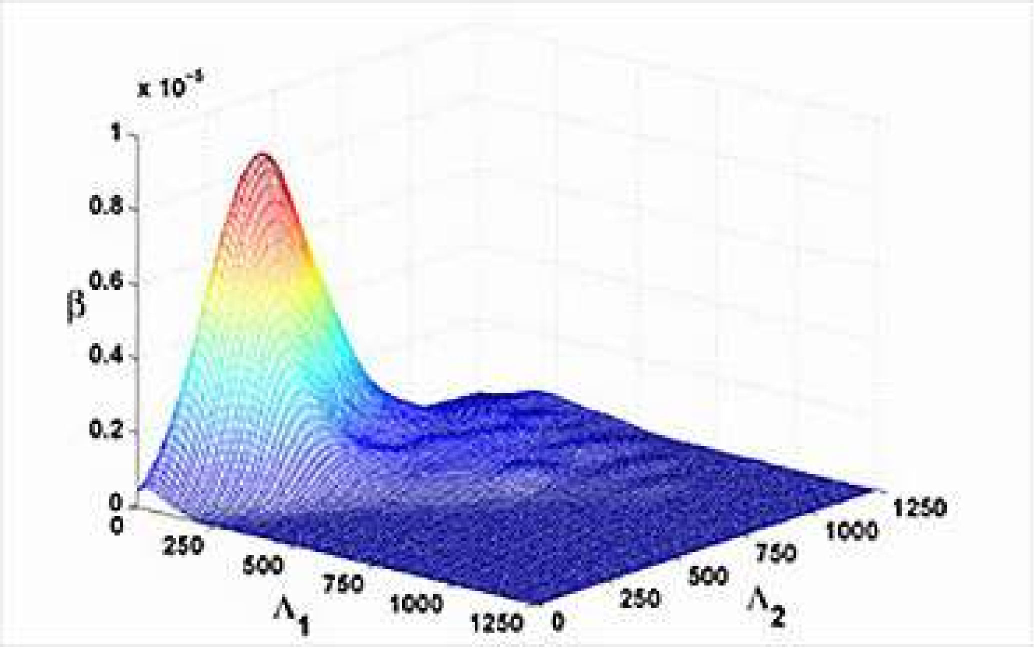
PSR J0348+0432 M=2.10+0.04−0.04 $M_\odot$
[Antoniadis et al. (2013)]

PSR J0952-0607 M= 2.35+0.17−0.17 $M_\odot$
[Romani et al. (2022) „Black Widow“ pulsar]

Λ_1-Λ_2 RELATION FROM GW170817

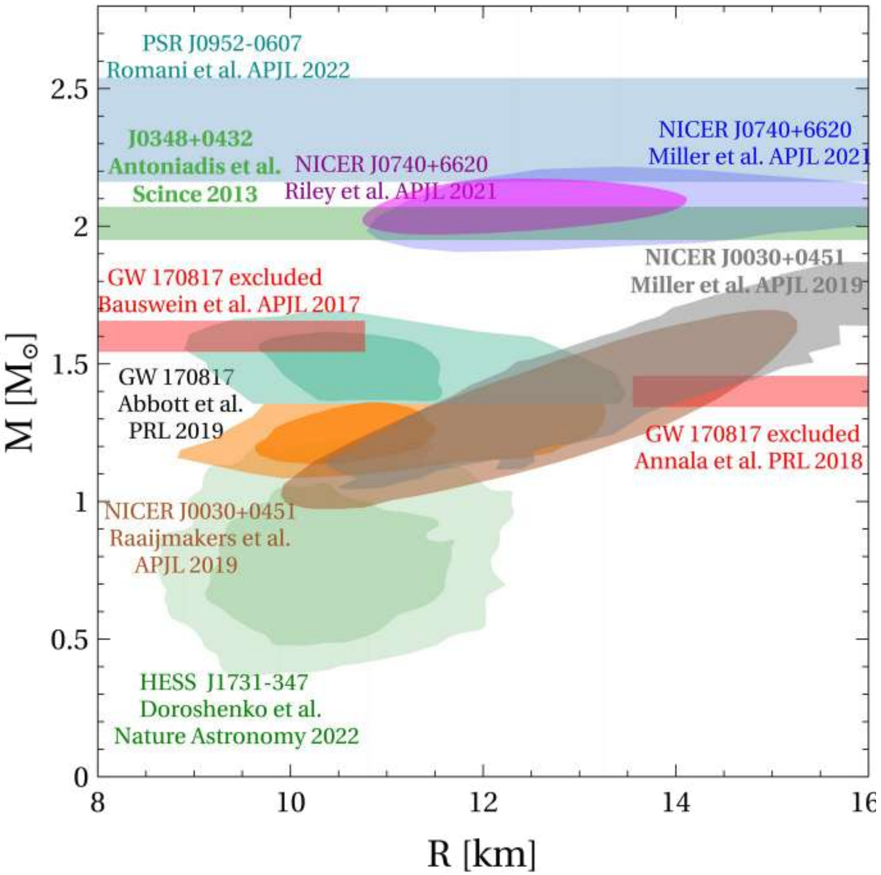
$$p(D_{GW}|(\eta_D,\eta_V)) = \int_{\varepsilon_c^{\min}}^{\varepsilon_c^{\max}(\eta_D,\eta_V)} f_{GW}(\Lambda_1(\varepsilon_c;\eta_D,\eta_V),\Lambda_2(\Lambda_1))\,pr(\eta_D,\eta_V)\,d\varepsilon_c$$





MASS-RADIUS CONSTRAINTS

$$p(D_{MR^{(i)}}|(\eta_D,\eta_V)) = \int_{\varepsilon_c^{\min}}^{\varepsilon_c^{\max}(\eta_D,\eta_V)} f_{MR}^{(i)}\big(M(\varepsilon_c;\eta_D,\eta_V),R(M)\big)\,pr(\eta_D,\eta_V)\,d\varepsilon_c$$



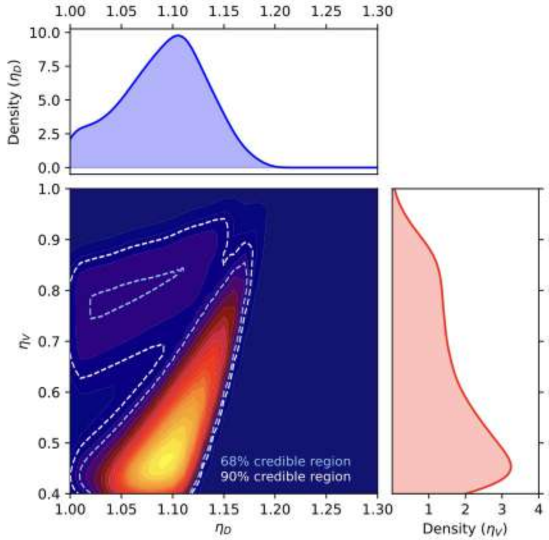
A) PSR J0030+0451: M= 1.44+0.07−0.07 $M_\odot$
and R= 13.7+2.6−1.5 km

(B) PSR J0740+6620: M= 2.08+0.07−0.07 $M_\odot$
and R= 13.7+2.6−1.5 km

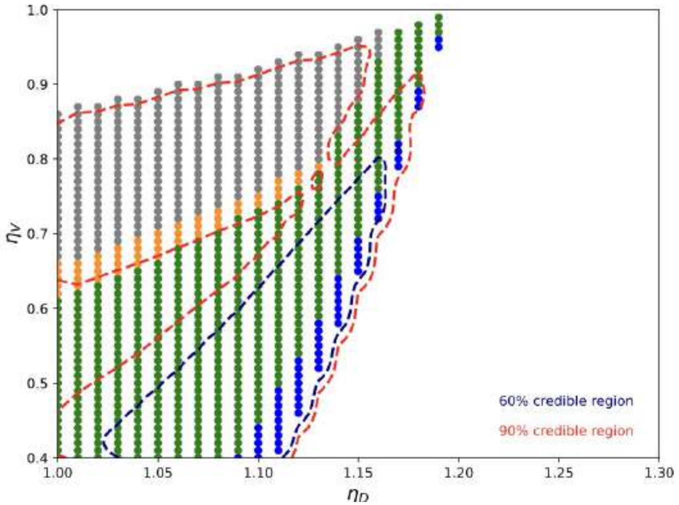
(C) PSR J0437-4715: M= 1.418+0.037−0.037 $M_\odot$
and R= 11.36+0.95−0.63 km

(D) HESS J1731-347: M= 0.77+0.20−0.17 $M_\odot$
and R= 10.04+0.86−0.78 km

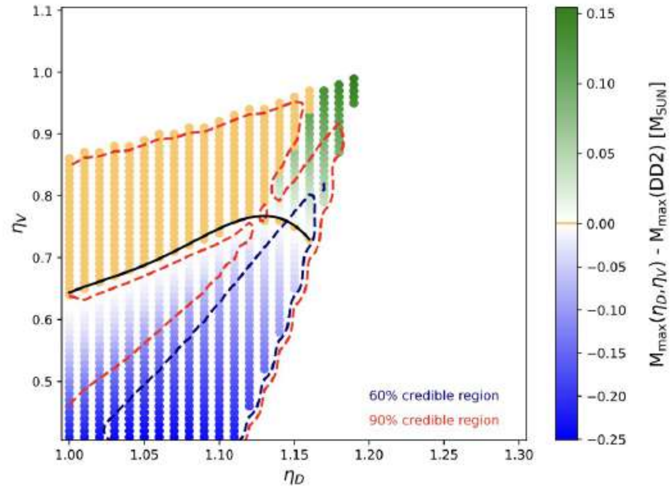
BA RESULTS



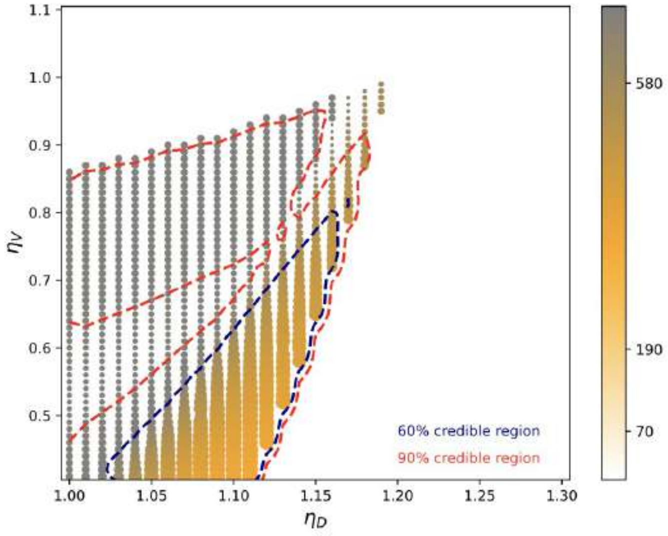
Density (η_D)
Density (η_V)
η_V
η_D
68% credible region
90% credible region
1.00 1.05 1.10 1.15 1.20 1.25 1.30
0.0 2.5 5.0 7.5 10.0
0.4 0.5 0.6 0.7 0.8 0.9 1.0
1 2 3 4



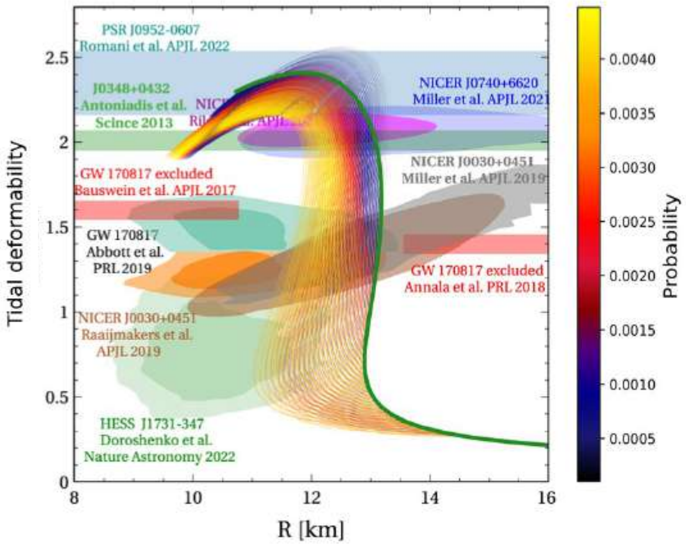
η_V
η_D
60% credible region
90% credible region
1.00 1.05 1.10 1.15 1.20 1.25 1.30
0.4 0.5 0.6 0.7 0.8 0.9 1.0



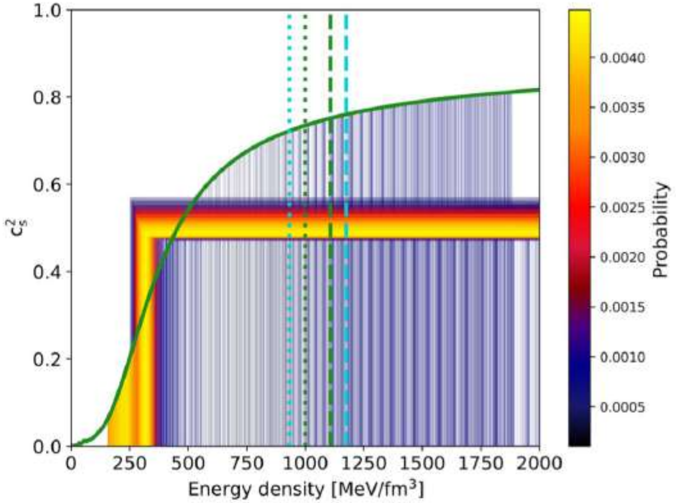
η_V
η_D
60% credible region
90% credible region
$M_{max}(\eta_D,\eta_V)$ - M_{max}(DD2) [M_{SUN}]
0.15 0.10 0.05 0.00 −0.05 −0.10 −0.15 −0.20 −0.25
1.00 1.05 1.10 1.15 1.20 1.25 1.30
0.5 0.6 0.7 0.8 0.9 1.0 1.1



η_V
η_D
60% credible region
90% credible region
580
190
70
1.00 1.05 1.10 1.15 1.20 1.25 1.30
0.5 0.6 0.7 0.8 0.9 1.0 1.1



PSR J0952-0607
Romani et al. APJL 2022
J0348+0432
Antoniadis et al.
Scince 2013
NICER J0740+6620
Miller et al. APJL 2021
NICER J0030+0451
Miller et al. APJL 2019
GW 170817 excluded
Bauswein et al. APJL 2017
GW 170817
Abbott et al.
PRL 2019
GW 170817 excluded
Annala et al. PRL 2018
NICER J0030+0451
Raaijmakers et al.
APJL 2019
HESS J1731-347
Doroshenko et al.
Nature Astronomy 2022
Tidal deformability
R [km]
Probability
8 10 12 14 16
0 0.5 1 1.5 2 2.5
0.0005 0.0010 0.0015 0.0020 0.0025 0.0030 0.0035 0.0040



c_s^2
Energy density [MeV/fm^3]
Probability
0 250 500 750 1000 1250 1500 1750 2000
0.0 0.2 0.4 0.6 0.8 1.0
0.0005 0.0010 0.0015 0.0020 0.0025 0.0030 0.0035 0.0040

HIC

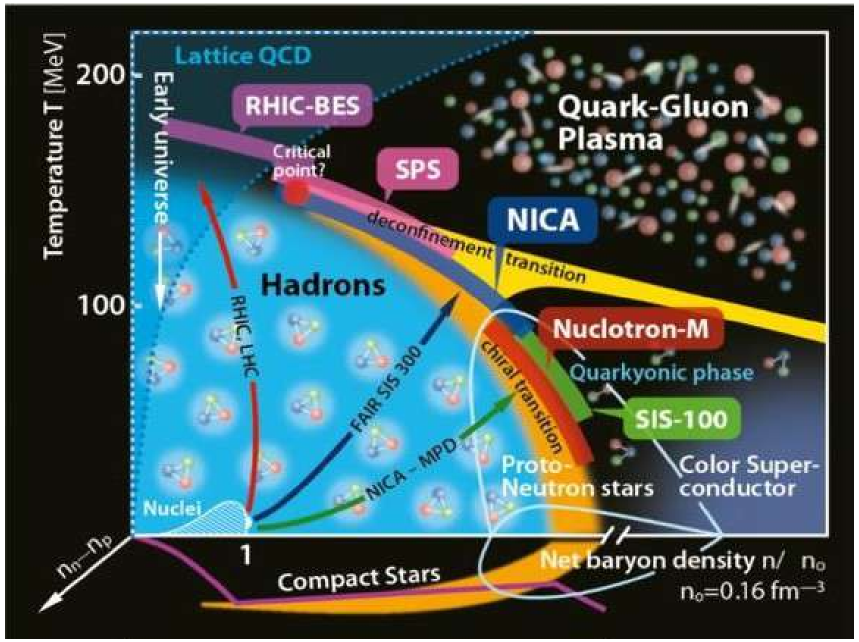
Data

For 2760 GeV and 5020 GeV: yields, pT-differential invariant yields, centrality dependence of mean pT, flow -> π±, K±, p, p-bar, v_2 etc.

Software: DUKE and/or JETSCAPE

Simulation of one event takes ~30 sec.

Parallel calculation has been applied!



To prove the principle, let us first try
to simultaneously analyze T=0 and mu=0.

THANK YOU FOR YOUR ATTENTION!

ELIZAVETA ZHEREBTSOVA

The primary task is to collect experimental data of key observational objects obtained from heavy ion collision experiments, especially those conducted at the Super Proton Synchrotron (SPS) within the NA61/SHINE experiment, in which the University of Wrocław is a participant, in order to compare them with simulation results.

Basic responsibilities:

- Collecting experimental data sets covering various observations, such as particle spectra, momentum analyses, particle production, etc.

- Extracting key observations from experimental data for comparison with data obtained from HIC simulations.

- Providing detailed reports describing methodologies, data sets used, and results of comparisons of experimental and simulation results.

BIPLAB MAHATO

The main task is to develop a unified approach to quark-hadron matter, in which hadrons appear as bound states that can undergo Mott dissociation when the density and/or temperature exceed critical values, and to demonstrate the Mott momentum phenomenon: a hadronic correlation, which is a continuous state of dissociation in the rest state in the medium, will become a true bound state when it moves with a momentum exceeding the Mott momentum.

Basic responsibilities:

- Derive the Beth-Uhlenbeck equation of state for quark-hadron matter from the NJL-type field model for interacting quark matter

- Determine the temperature and density conditions for which this model transforms into the Walecka model

- Obtain the Mott momentum for nucleons as a function of density and temperature and discuss similarities with the quarkyonic matter model.

Observables in ALICE for the work

2760 GeV:

- Yields:
    - ☐ Particles: $\pi^{\pm}$, $K^{\pm}$, p, $\bar{p}$, available centralities: 0-80% with 5% step
    - ☐ φ with centralities 0-90%, step 5% for 0-10 and 10% for others
    - ☐ K* with centralities 0-60% with the step 20%
    - ☐ Ks, Λ with centralities 0-90%, step 5% for 0-10 and 10% for others
- p_T -differential invariant yields
    - ☐ $\pi^{\pm}$, available centralities: 0-60% with 5% step
    - ☐ p, $\bar{p}$, available centralities: 0-10% with 5% step
    - ☐ φ, available centralities: 0-90% with 5% step
    - ☐ K*, available centralities: 0-90% with 5% step
    - ☐ Xi & Omega, available centralities: 0-60% with 10% step
- Centrality dependence of mean p_T
    - ☐ Particles: $\pi^{\pm}$, $K^{\pm}$, p, $\bar{p}$
    - ☐ φ and K*, centralities as for yields
- Flow
    - ☐ v_2 integrated over the p_T range $0.2 < \mathrm{pT} < 5.0$ GeV, $2\ or\ 4$ cumulants method for charged particles, centrality 0-80% with 5% step
    - ☐ Transverse momentum dependence of $\mathrm{v}_2\,\{4\}$, centrality 20-40% with step 10%
    - ☐ Pseudorapidity dependence of $\mathrm{v}_2\,\{2\}$, $\mathrm{v}_3\,\{2\}$, $\mathrm{v}_4\,\{2\}$, $\mathrm{v}_2\,\{4\}$, centrality 0-5%, 5-10% and 10-80% with step 10%