# Chemical Freeze-Out in the QCD Phase Diagram

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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**Wroclaw, IFT Seminar, 24.01.2025**

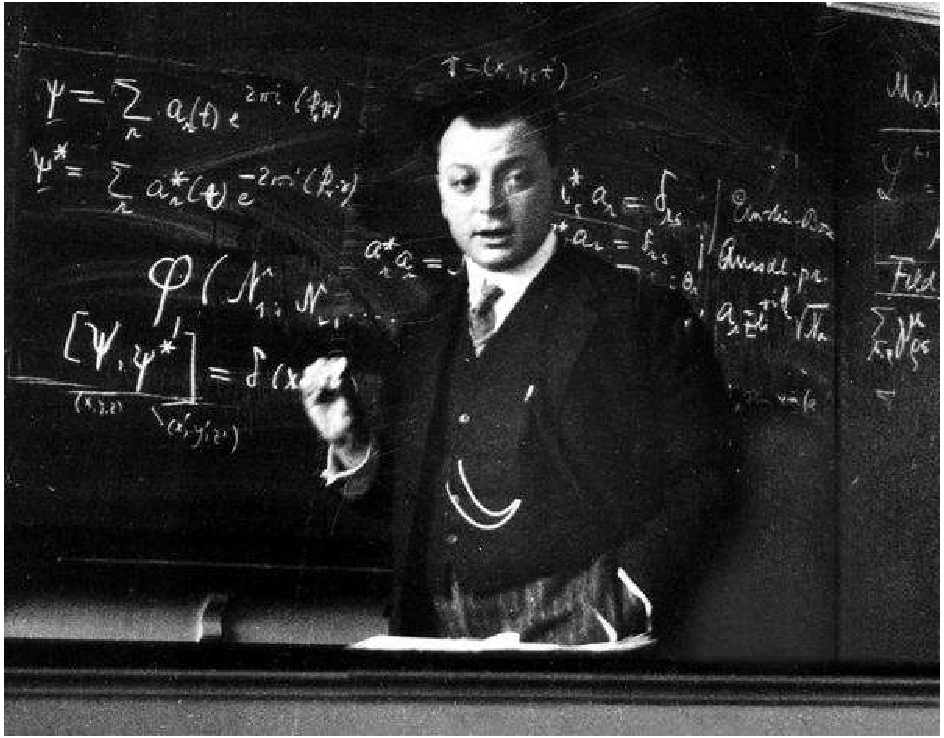
# 100 years of the Pauli Exclusion Principle

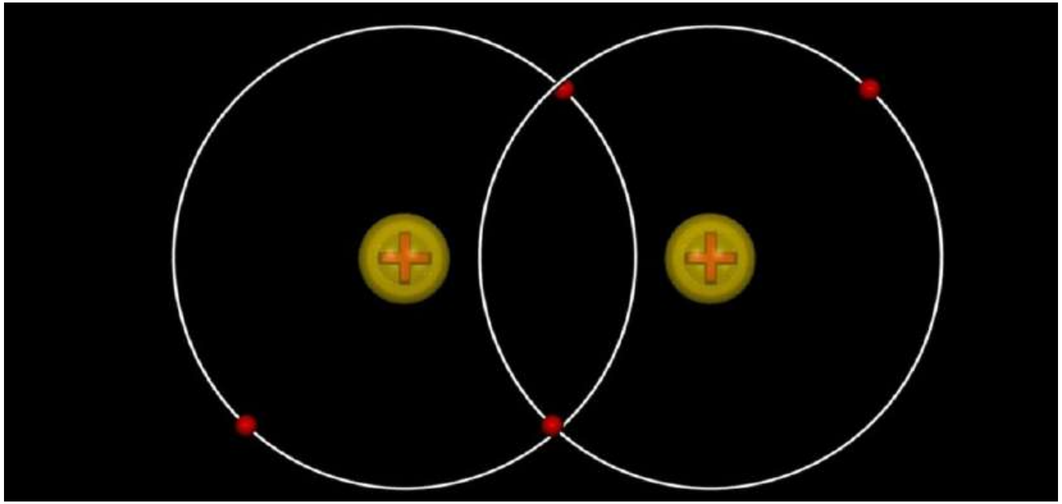
**Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren.**

Von **W. Pauli** jr. in Hamburg.

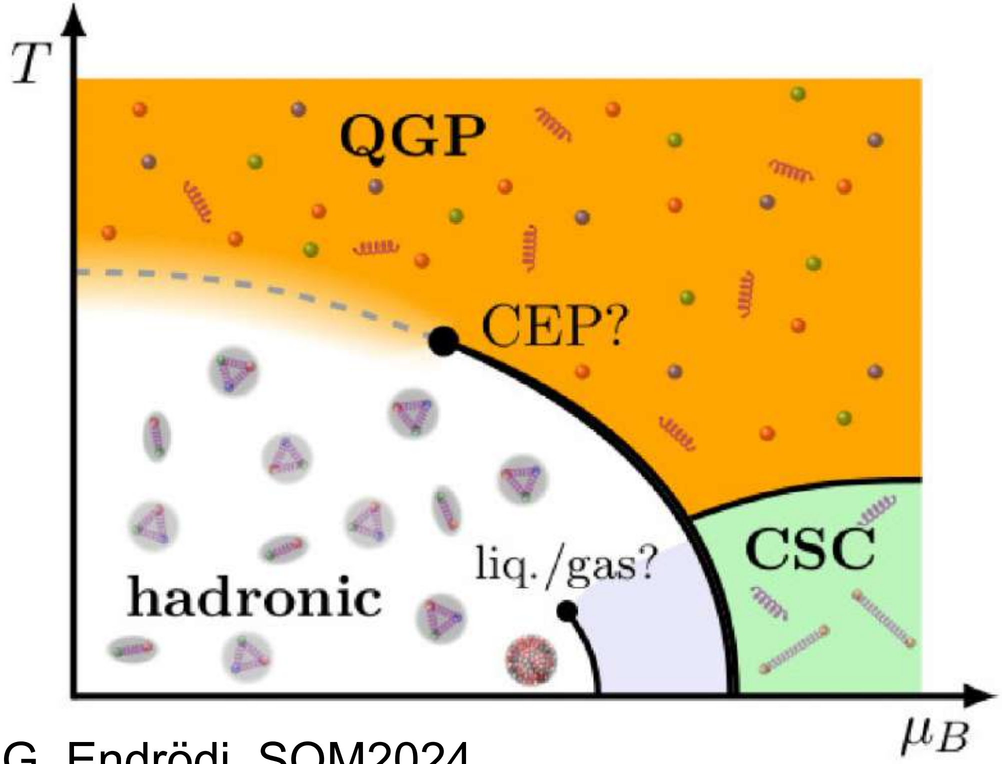
(Eingegangen am 16. Januar 1925.)

Zeitschrift für Physik. Bd. XXXI.





# Exploring the QCD Phase Diagram




G. Endrödi, SQM2024

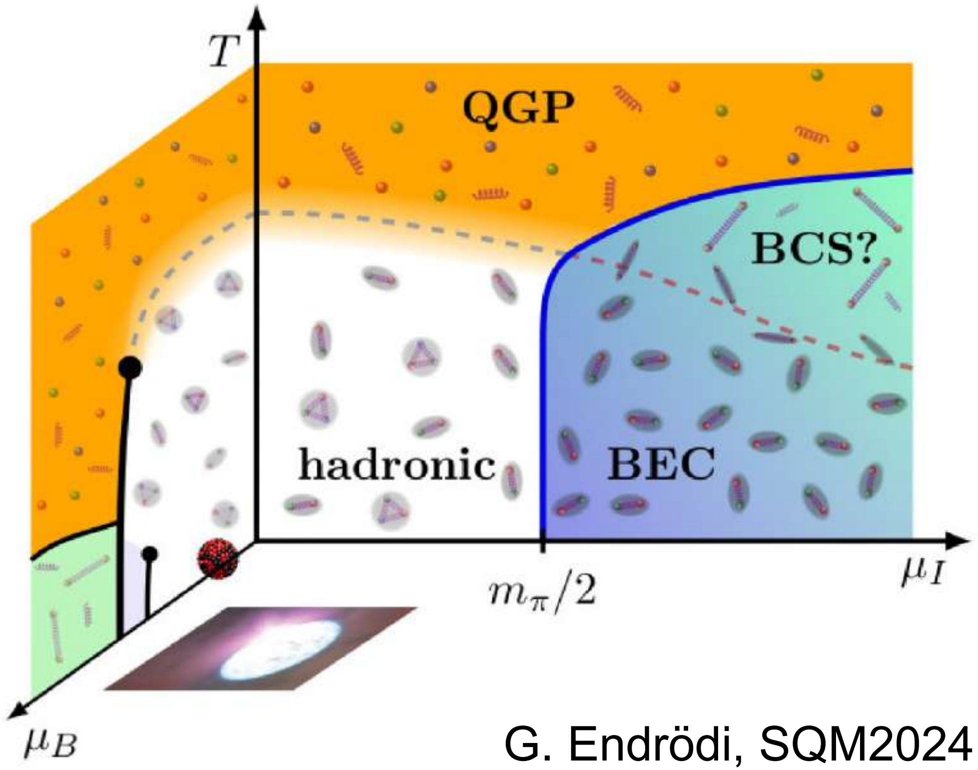
The Goal: Proving the phase structure!

Lattice QCD results only for pseudocritical temperature Tc near μ ~ 0 (sign problem)

Liquid-gas PT indicated in experiment

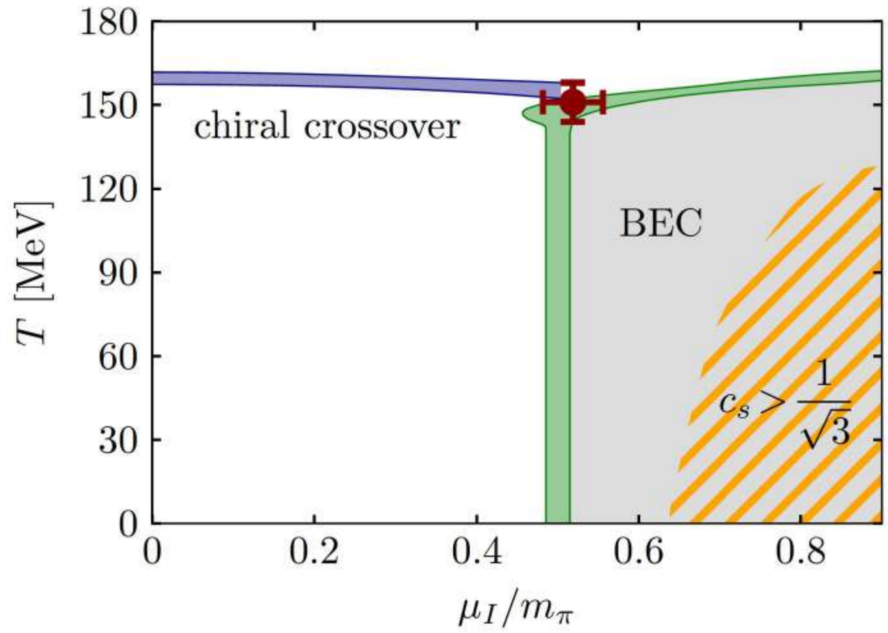
Other structures are so far model dependent conjectures!!

# Exploring the QCD Phase Diagram




G. Endrödi, SQM2024

Isospin-QCD ? Lattice QCD results !!




Brandt, Endrödi, Schmalzbauer, PRD 97, 054514 (2018)

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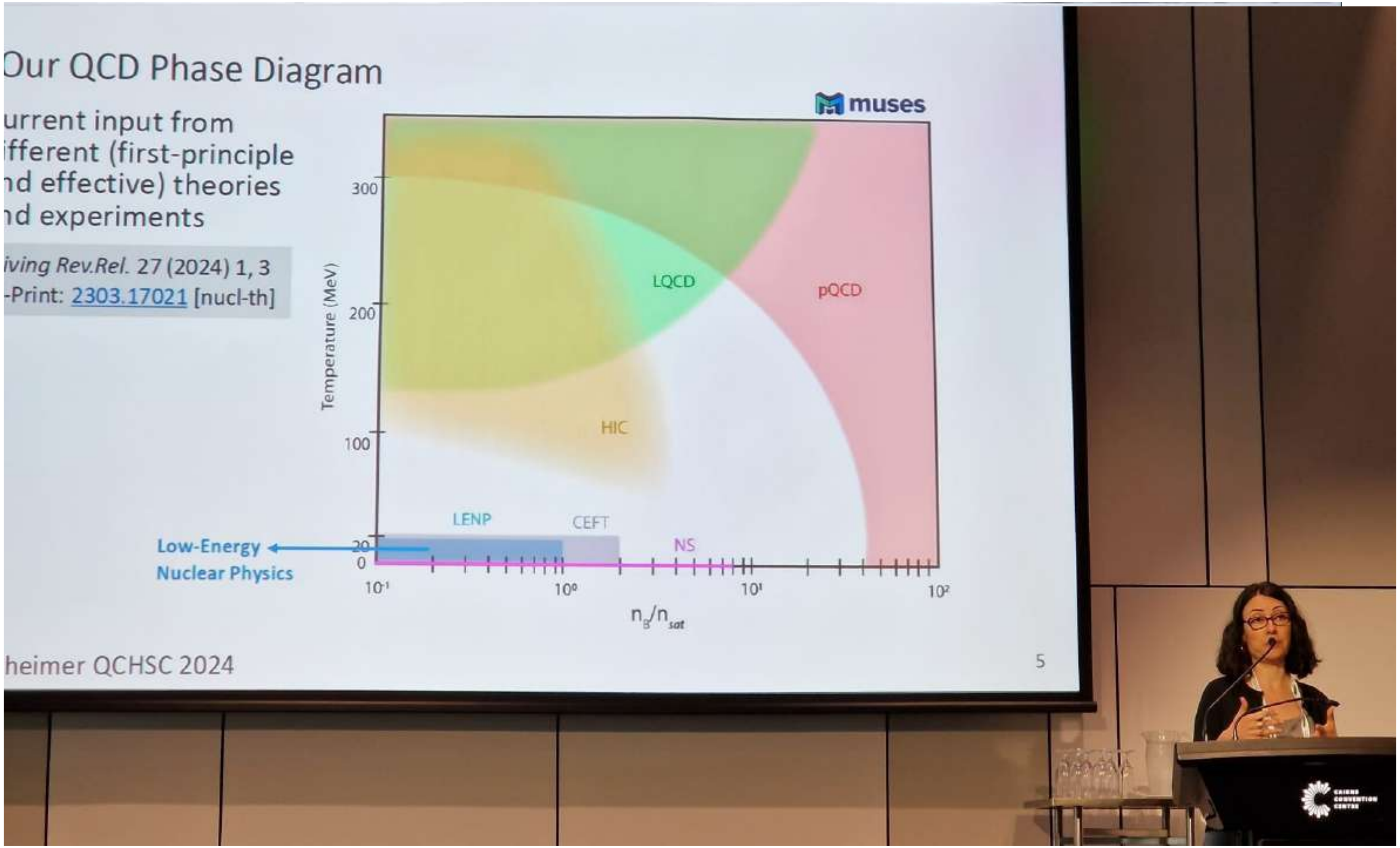



V. Dexheimer, QCHS2024

# Exploring the QCD Phase Diagram

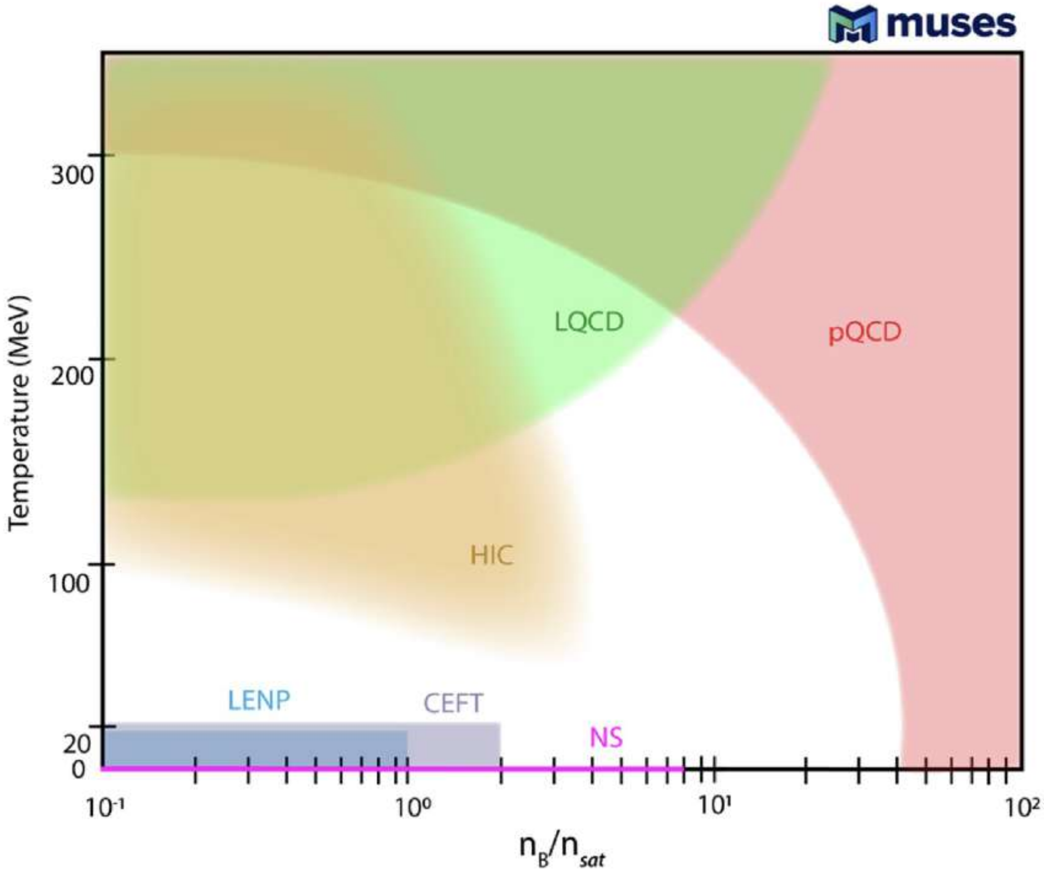
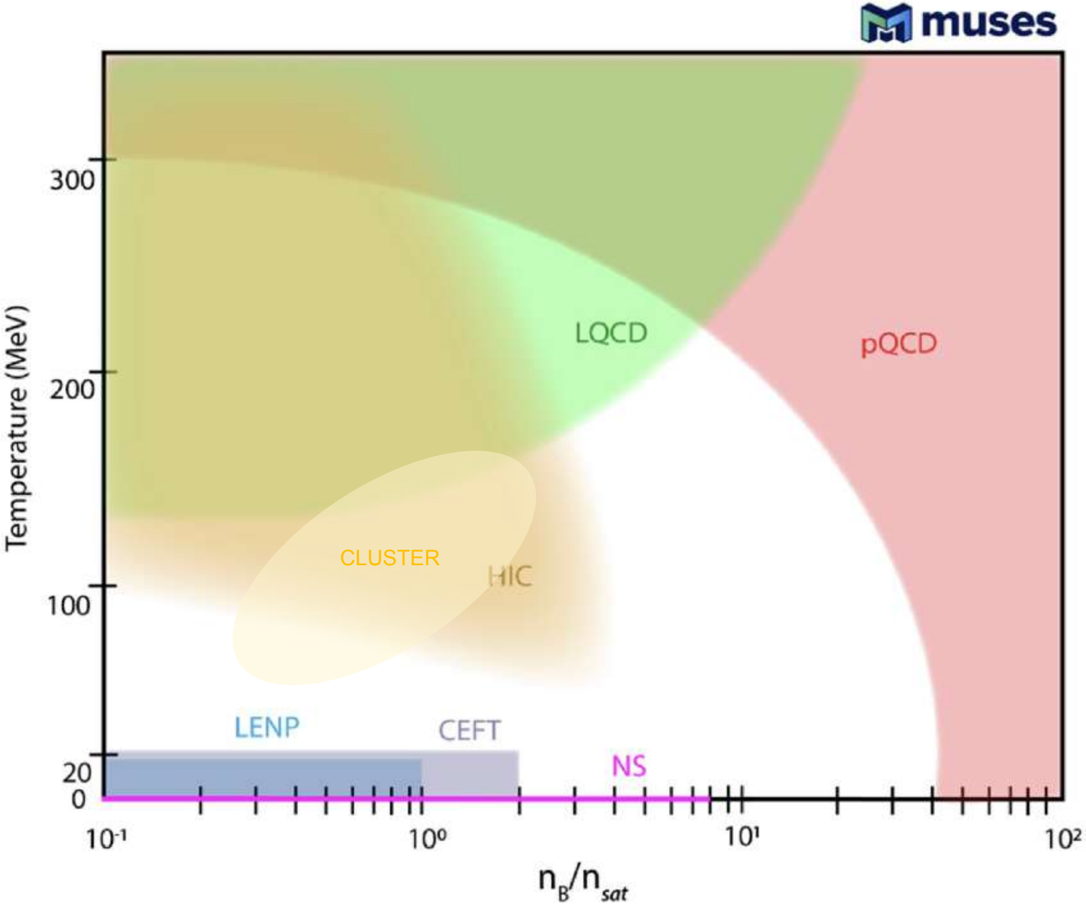



**Fig. 1** Regions of the QCD phase diagram where constraints from heavy-ion collisions (HIC), lattice QCD (LQCD), perturbative QCD (pQCD), low-energy heavy-ion collisions (LENP), chiral effective field theory (χEFT), and astrophysics (neutron stars, NS) are available

Living Reviews in Relativity (2024)27:3
https://doi.org/10.1007/s41114-024-00049-6

# Exploring the QCD Phase Diagram

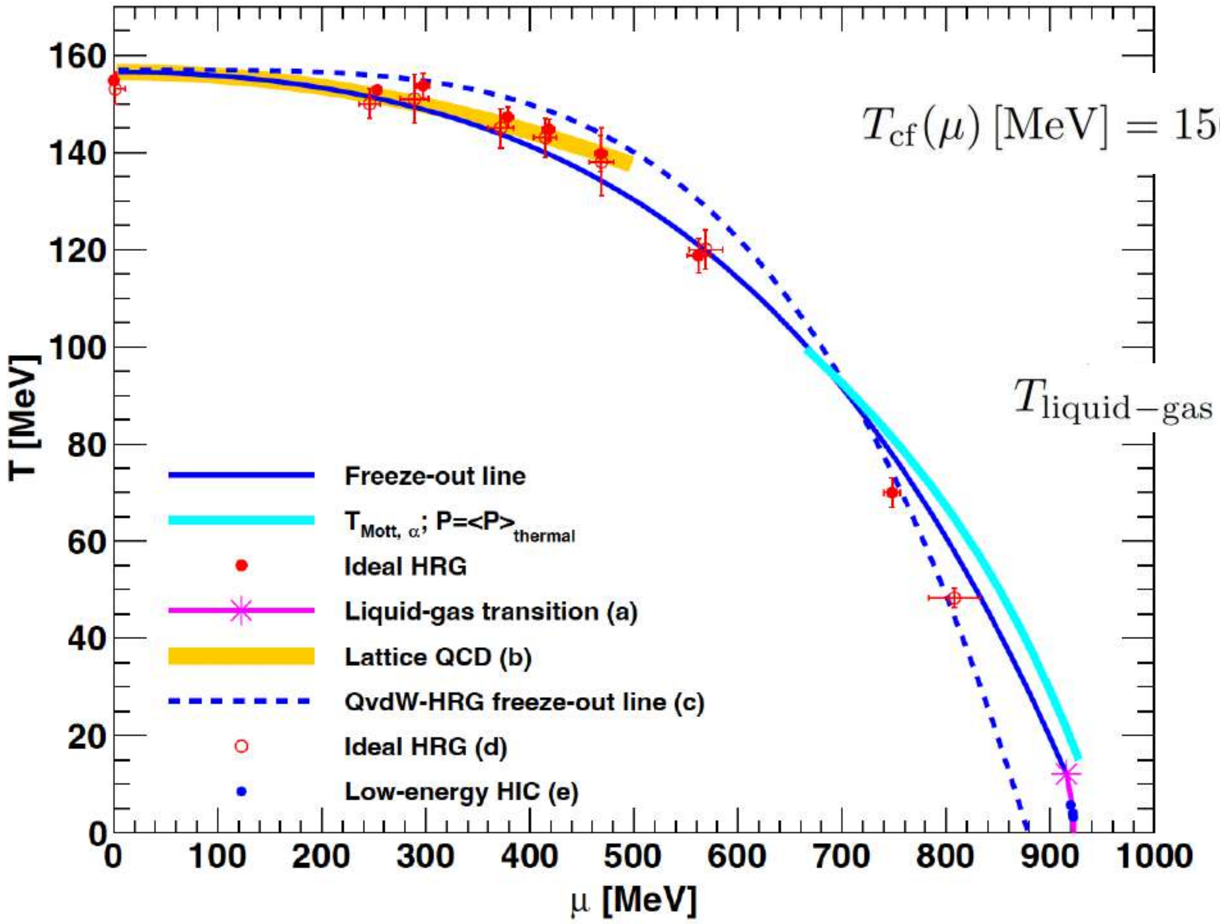



**This Talk:**

Chemical Freeze-Out @ T ~ 20 – 100 MeV

- Statistical Model Fit, T – μ diagram
- CFO in the T – n diagram
- Mott dissociation for light clusters
- CFO as inverse Mott dissociation
- Summary & Outlook

# Statistical Model Fit for CFO, T – μ Diagram




The freeze-out line (blue):

$$T_{\rm cf}(\mu)\,[\text{MeV}] = 156.5 - 76.68\,(\mu\,[\text{GeV}])^2 - 139.7\,(\mu\,[\text{GeV}])^4$$

interpolates between lattice QCD chiral restoration crossover (yellow) and CEP of the liquid-gas transition

$$T_{\text{liquid-gas}} = 12.1\,\text{MeV},\ \mu_{\text{liquid-gas}} = 915.61\,\text{MeV}$$

Mott dissociation line for alphas is well correlated in baryon-dominant region

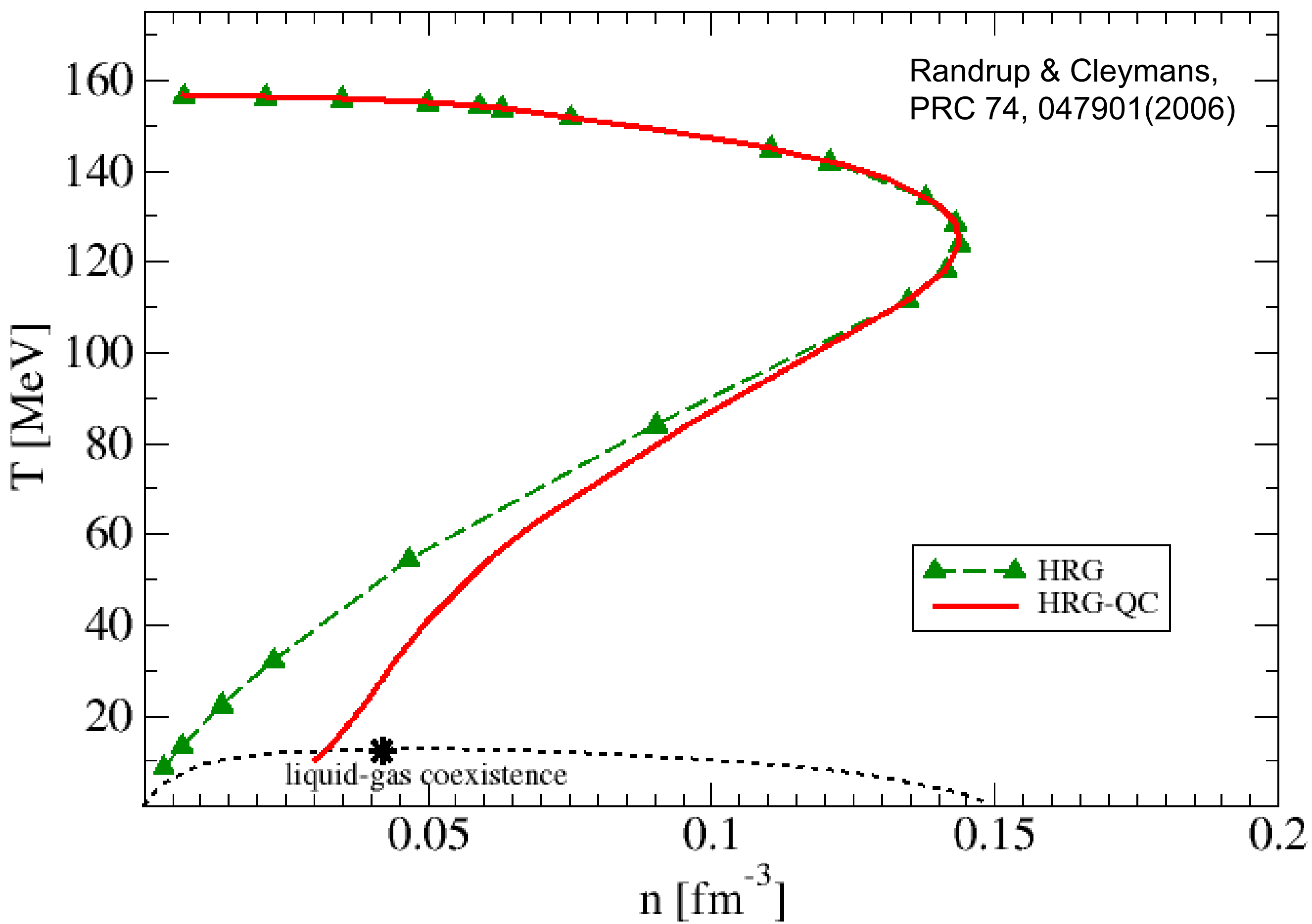
(a ) Typel et al., PRC 81, 015803 (2010)

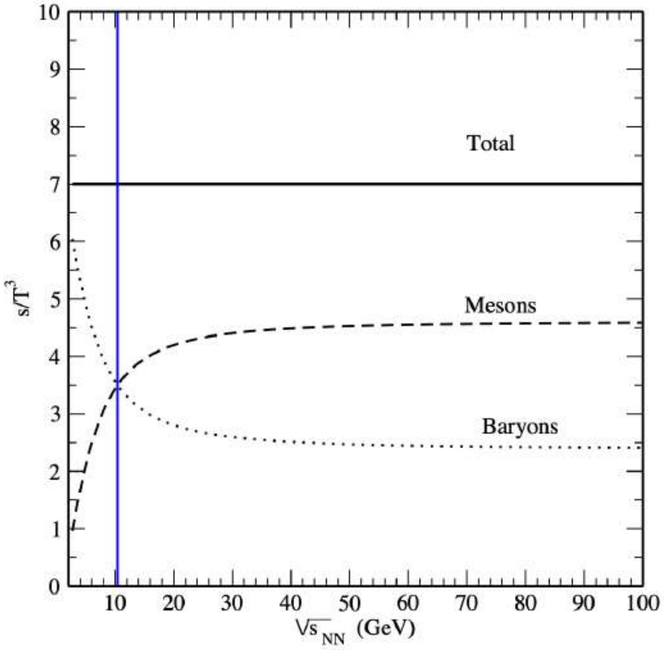
(c ) Poberezhnyuk et al., PRC 100, 054904 (2019)

(e ) Natowitz et al., PRL 104, 202501 (2010)

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

# Statistical Model Fit for CFO, T – n Diagram

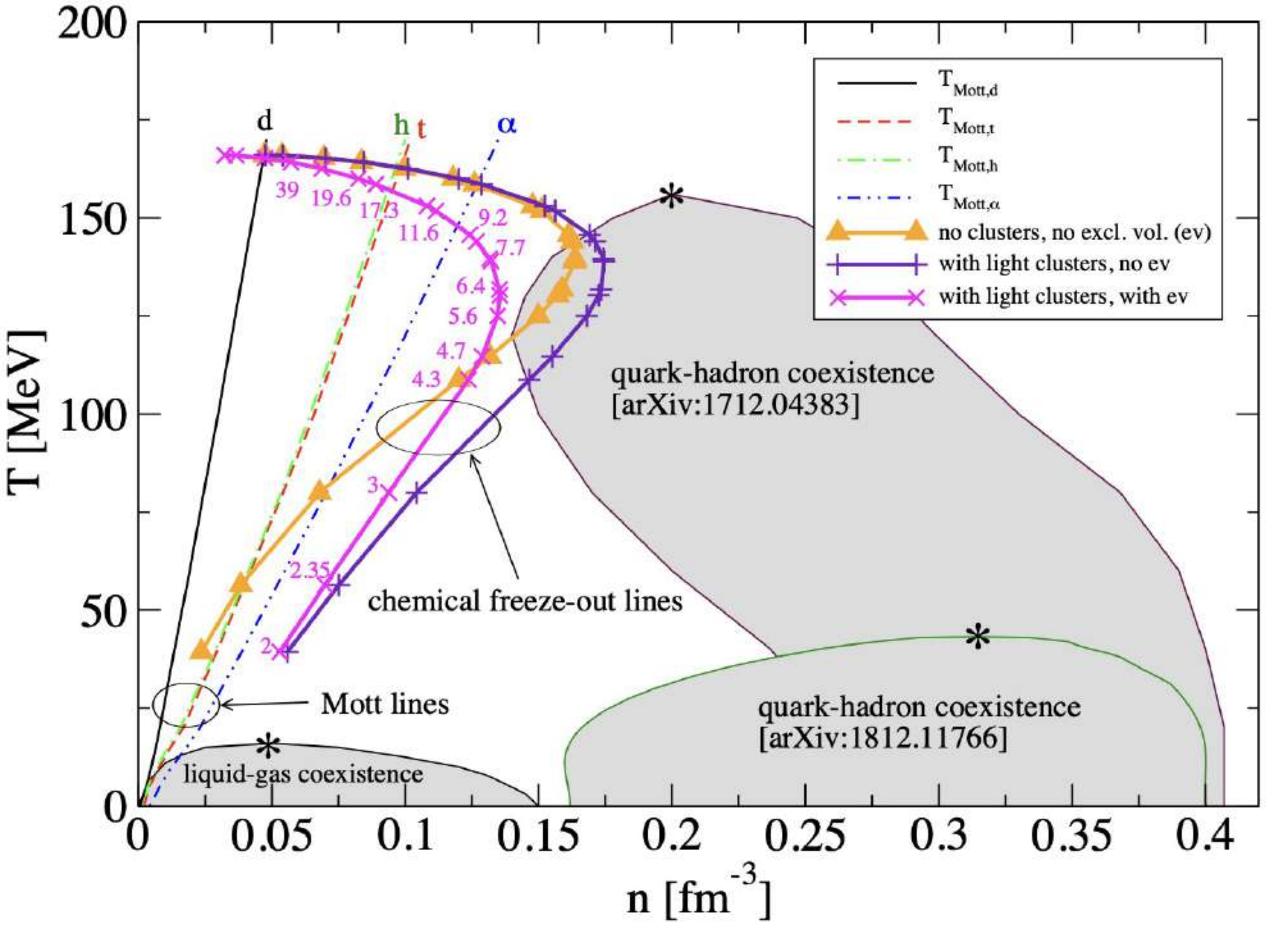



Transition meson- to baryon-dominance at T ~ 140 MeV

Andronic et al., NPA 837, 65 (2010)

# Statistical Model Fit for CFO, T – n Diagram




Correlation of CFO line with chiral restoration/ deconfinement gets lost at T < 140 MeV

Is CFO in the baryon-dominated region correlated with Mott lines for dissociation of light clusters?

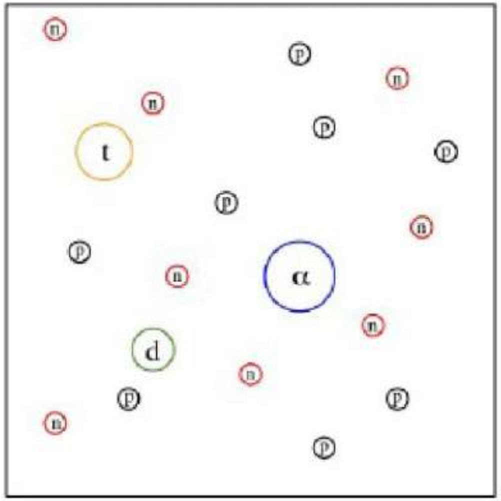
Blaschke et al., Springer Proc. Phys. 250, 183 (2020); arXiv:2001.02156 (SQM 2019)

# Mott dissociation for bound states in a plasma

Chemical picture:

Ideal mixture of reacting components

Mass action law

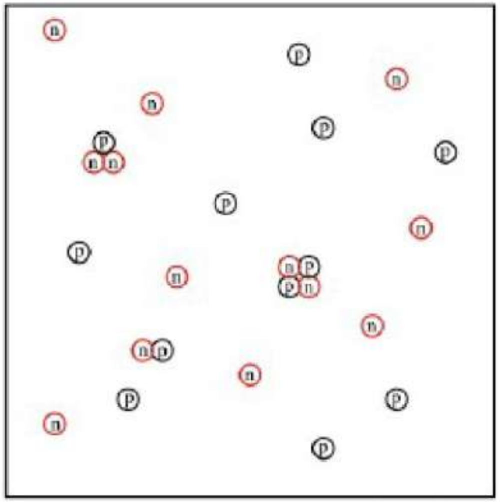



Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction




Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

# Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$= E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

# Mott dissociation for bound states in a plasma

## Effective wave equation for deuterons in nuclear matter

[Derivation of a „Plasma Hamiltonian“ from a Bethe-Goldstone Eq. for two-particle states]

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

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[R. Zimmermann et al. (5-men-work), Phys. Stat. Sol. (b) 90 (1978) 175]

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Nuclear Physics A379 (1982) 536–552


**PARTICLE CLUSTERING AND MOTT TRANSITIONS IN NUCLEAR MATTER AT FINITE TEMPERATURE**
**(I). Method and general aspects**

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L. MÜNCHOW

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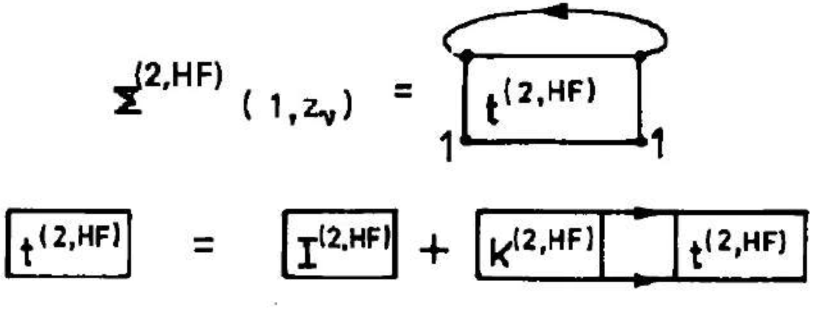

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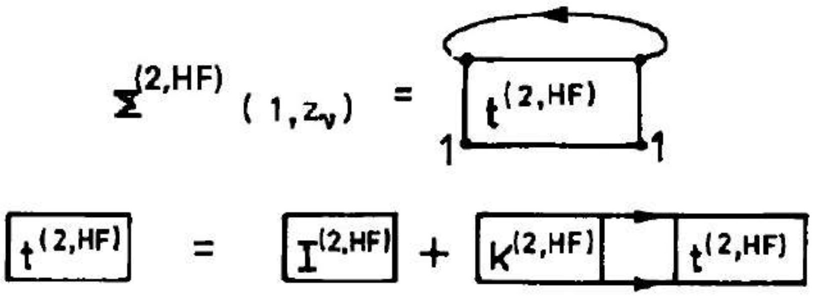


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$$\times(\hbar\Omega_\lambda-E(1)-\Delta^{\mathrm{HF}}(1)-E(2)-\Delta^{\mathrm{HF}}(2))$$
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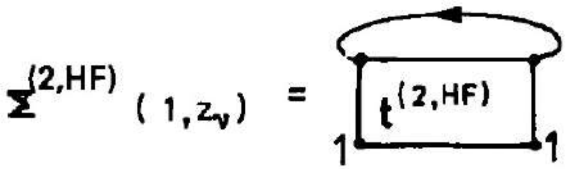


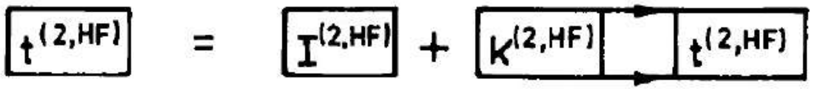


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$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12)+\sum_{1'2'}V(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=\sum_{1'2'}\{\tfrac{1}{2}(f(1)+f(2)+f(1')+f(2'))V(12,1'2')$$
$$-(\Delta^{\mathrm{HF}}(1)+\Delta^{\mathrm{HF}}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=-\sum_{1'2'}H^{(2,\mathrm{HF})}_{\mathrm{nucl.\,matter}}(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2').$$

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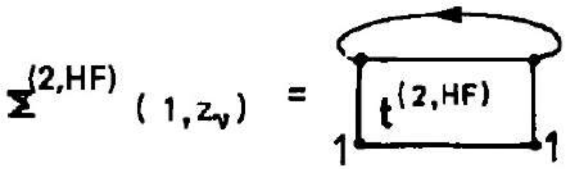


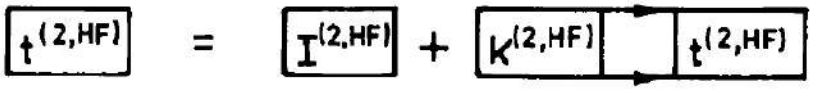


$$t^{(2,\mathrm{HF})}(1234,\Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}} - \hbar\Omega_\lambda}\left(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4))\right)$$
$$\times(\hbar\Omega_\lambda - E(1) - \Delta^{\mathrm{HF}}(1) - E(2) - \Delta^{\mathrm{HF}}(2))$$
$$\times(E_\alpha^{\mathrm{HF}} - E(3) - \Delta^{\mathrm{HF}}(3) - E(4) - \Delta^{\mathrm{HF}}(4))\,,$$

$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12) + \sum_{1'2'} V(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$
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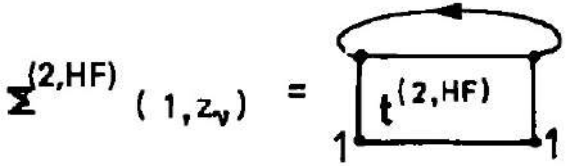
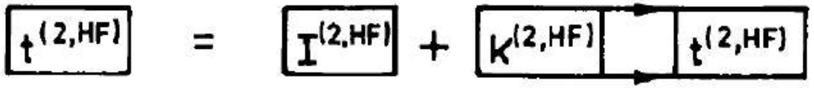
$$\Delta E^{\mathrm{HF}}_{\mathrm{d},P} = \tfrac{3}{2}t_0\rho_{\mathrm{nucl}} - \sqrt{2}t_0\rho_{\mathrm{nucl}}(1+x_0)\left(1+\frac{\pi}{\alpha^2\Lambda^2}\right)^{-3/2}\exp\left[-\frac{P^2}{16\alpha^2}\left(1+\frac{\pi}{\Lambda^2\alpha^2}\right)^{-1}\right]$$

$$= 2\Delta^{\mathrm{HF}}(0) + \Delta^{\mathrm{Pauli}}_{\mathrm{d},P}\rho_{\mathrm{nucl}}, \qquad (2.$$

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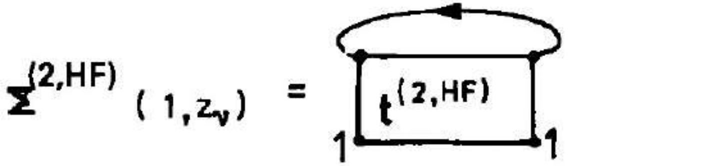
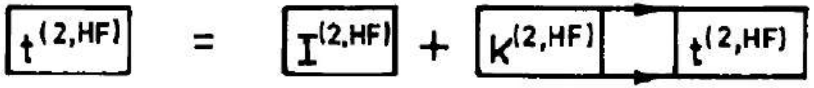
$$= 2\Delta^{\mathrm{HF}}(0) + \Delta^{\mathrm{Pauli}}_{\mathrm{d},P}\,\rho_{\mathrm{nucl}}\,, \qquad \alpha = \hbar(|E_{\mathrm{d}}^0|/M)^{1/2} = 0.23\ \mathrm{fm}^{-1}\,,$$

$$\Lambda = (2\pi\beta\hbar^2/M)^{1/2}$$

$$\rho^{(0)}_{\mathrm{nucl}}(\beta,\mu) = \frac{4}{\Lambda^3}\exp(\beta\mu)\,, \quad t_0 = -1057.3\ \mathrm{MeV\cdot fm^3}\ \text{and}\ x_0 = 0.56.$$

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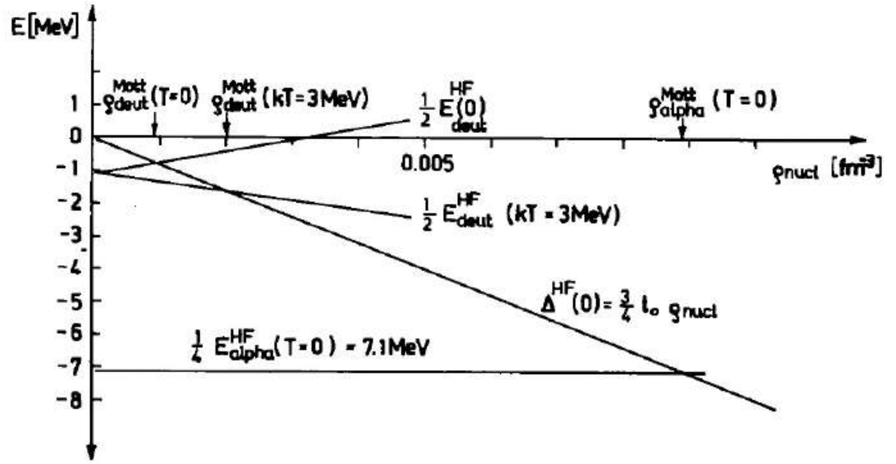
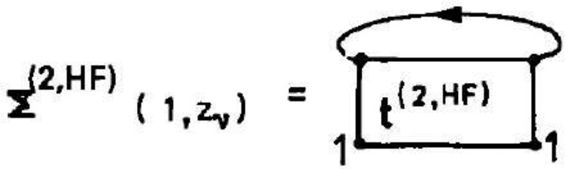
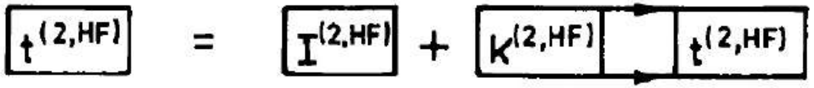
# Mott dissociation for clusters in nuclear matter

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$$=2\Delta^{HF}(0)+\Delta^{\text{Pauli}}_{d,P}\rho_{nucl},$$

$$\alpha=\hbar(|E_d^0|/M)^{1/2}=0.23\ \text{fm}^{-1}.$$
$$\Lambda=(2\pi\beta\hbar^2/M)^{1/2}$$

$$\{E(1)+E(2)-E_\alpha^{HF}\}\phi_\alpha^{HF}(12)+\sum_{1'2'}V(12,1'2')\phi_\alpha^{HF}(1'2')$$
$$=\sum_{1'2'}\{\tfrac{1}{2}(f(1)+f(2)+f(1')+f(2'))V(12,1'2')$$
$$-(\Delta^{HF}(1)+\Delta^{HF}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{HF}(1'2')$$
$$=-\sum_{1'2'}H^{(2,HF)}_{\text{nucl. matter}}(12,1'2')\phi_\alpha^{HF}(1'2').$$

$$\rho^{(0)}_{nucl}(\beta,\mu)=\frac{4}{\Lambda^3}\exp(\beta\mu),\quad t_0=-1057.3\ \text{MeV}\cdot\text{fm}^3\ \text{and}\ x_0=0.56.$$

# Mott dissociation for clusters in nuclear matter

Present state-of-the-art:

G. Röpke, Nuclear matter equation of state including two-, three-, and four-nucleon correlations, Phys. Rev. C 92 (5) (2015) 054001, https://doi.org/10.1103/PhysRevC.92.054001, arXiv:1411.4593.

$$E_{A,\nu}(P) = E^0_{A,\nu}(P) + \Delta E^{\rm SE}_{A,\nu}(P) + \Delta E^{\rm Pauli}_{A,\nu}(P) + \Delta E^{\rm Coulomb}_{A,\nu}(P).$$

The cluster binding energies $E^{\rm bind}_{A,\nu}(P;T,n_B,Y_p,T_{\rm eff})$ are defined as

$$E^{\rm bind}_{A,\nu}(P;T,n_B,Y_p) = -[E_{A,\nu}(P;T,n_B,Y_p) - E^{\rm cont}_{A,\nu}(P;T,n_B,Y_p)]$$

$$E^{\rm cont}_{A,\nu}(P;T,n_B,Y_p) = N E_n(P/A;T,n_B,Y_p) + Z E_p(P/A;T,n_B,Y_p)$$

the in-medium dispersion relations for nucleons ($\tau = n, p$) are defined as

$$E_\tau(p;T,n_B,Y_p) = \sqrt{[m_\tau c^2 - S(T,n_B,Y_p)]^2 + \hbar^2 c^2 p^2} + V_\tau(T,n_B,Y_p) - m_\tau c^2$$

$$S_i(T,n_B,Y_p) = (4463 - 6.610\,T - 0.1703\delta^2 + 4.112\delta^4)n_B \times \frac{1 + c_1 n_b + c_2 n_B^2}{1 + c_3 n_b + c_4 n_B^2},$$

$$V_p(T,n_B,Y_p) = (3403 + 0.000052\,T - 486.6\,\delta - 2.420\,\delta^2)n_B \times \frac{1 + d_1 n_b + d_2 n_B^2}{1 + d_3 n_b + d_4 n_B^2}$$

Scalar and vector mean fields from RDF EoS DD2
S. Typel et al., Phys. Rev. C 81, 015803 (2010)

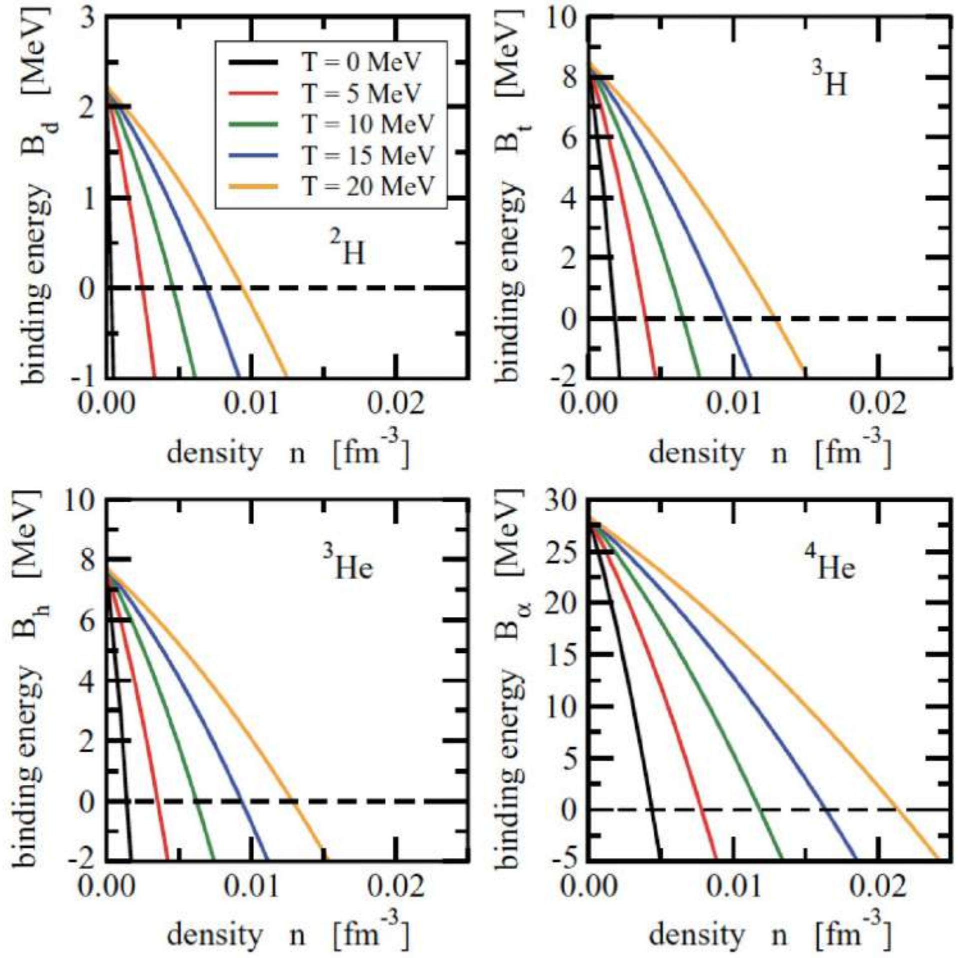
Parameter fit provided by G. Röpke et al., Phys. Part. Nucl. Lett. 15, 225 (2018)

$$c_1 = 20.56 - 0.04099\,T - 0.3394\,\delta^2 + 0.9972\,\delta^4,$$
$$c_2 = 15.98 + 0.8664\,T - 2.020\,\delta^2 - 3.018\,\delta^4,$$
$$c_3 = 24.27 - 0.07417\,T - 0.5427\,\delta^2 + 1.196\,\delta^4,$$
$$c_4 = 114.6 + 1.350\,T + 2.674\,\delta^2 + 0.7268\,\delta^4,$$

$$d_1 = 0.6629 - 0.006142\,T - 1.141\,\delta - 0.7176\,\delta^2,$$
$$d_2 = 10.7780 + 0.004432\,T + 0.8020\,\delta + 0.4576\,\delta^2,$$
$$d_3 = 3.433 + 0.000104\,T - 1.549\,\delta - 0.3360\,\delta^2,$$
$$d_4 = 23.01 - 0.03302\,T - 5.923\,\delta + 0.05090\,\delta^2,$$

with the isospin asymmetry $\delta = (1 - 2Y_p)$

# Binding energies for light clusters in T – n plane



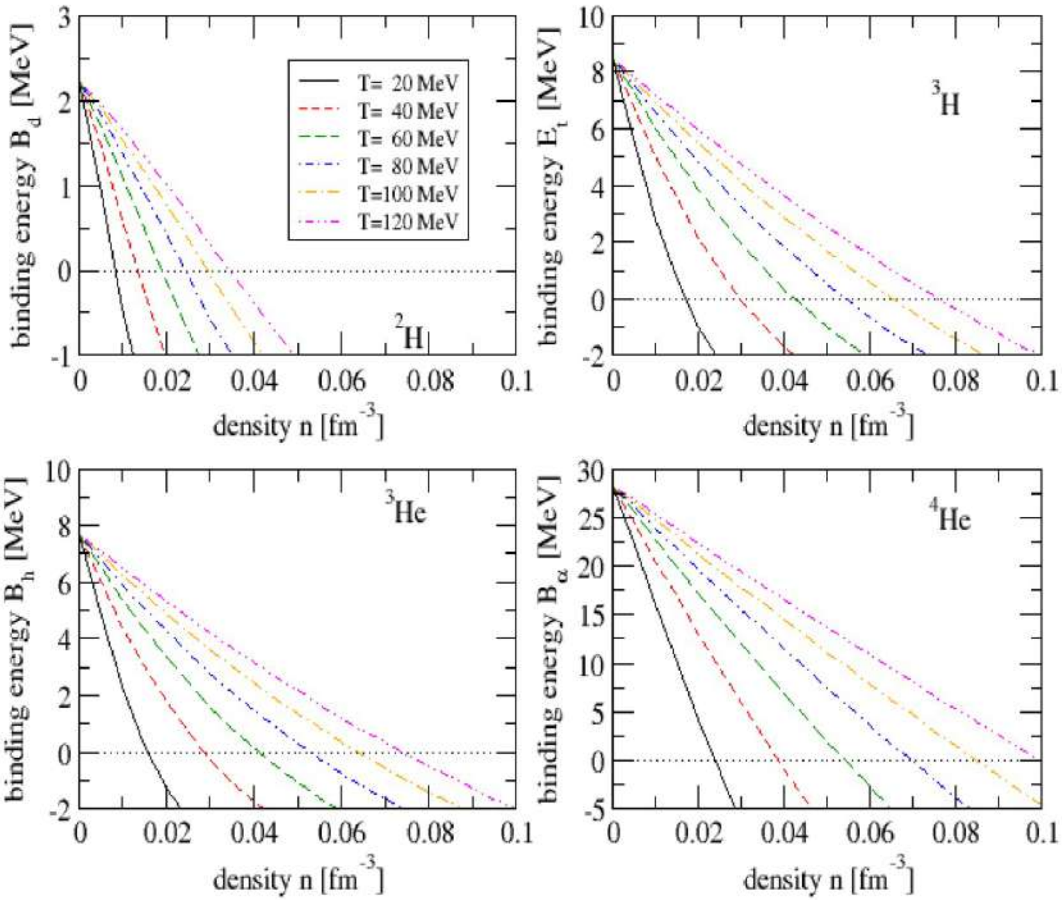

Vanishing binding energies
Indicate Mott effect for the
Light clusters!

Mott-lines in the T-μ plane
can be extracted, where the
Binding energy vanishes

Here lower temperatures:
0 < T[MeV] < 20

S. Typel et al., PRC 81, 015803 (2010)

# Binding energies for light clusters in T – n plane

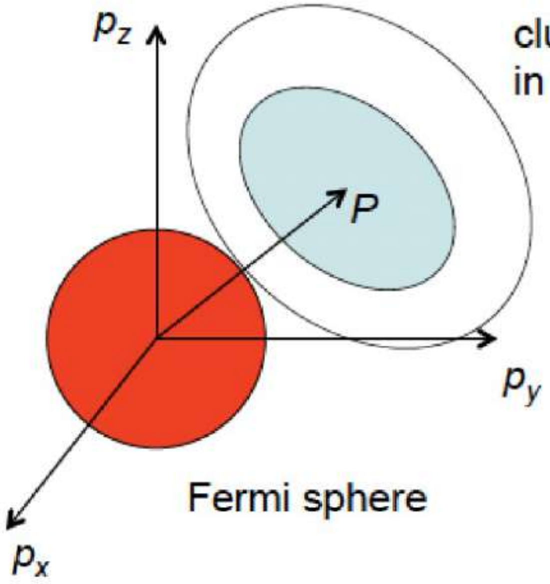



Mott-lines in the
T-μ plane can be
extracted, where
The binding
energy vanishes

Here higher
temperatures:

20 < T[MeV] < 120

# Pauli blocking: phase space occupation




cluster wave function (deuteron, alpha,…)
in momentum space

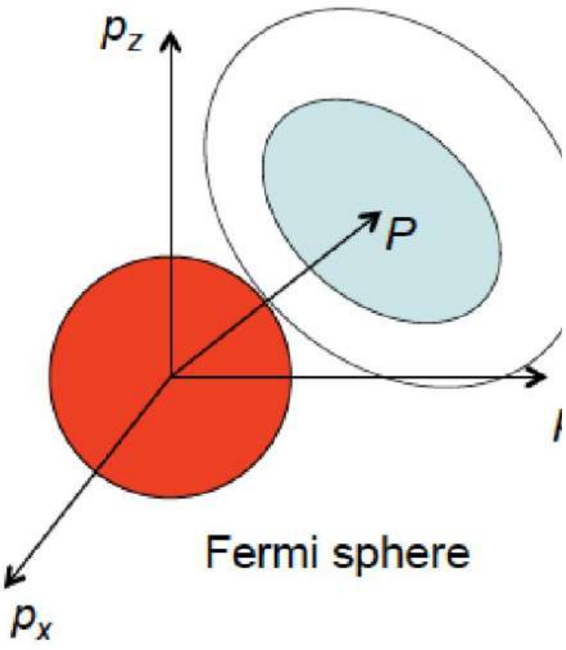
$P$ - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum $P$

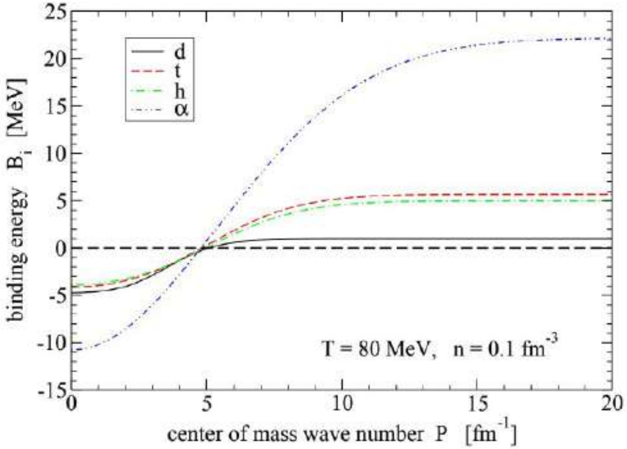
momentum space

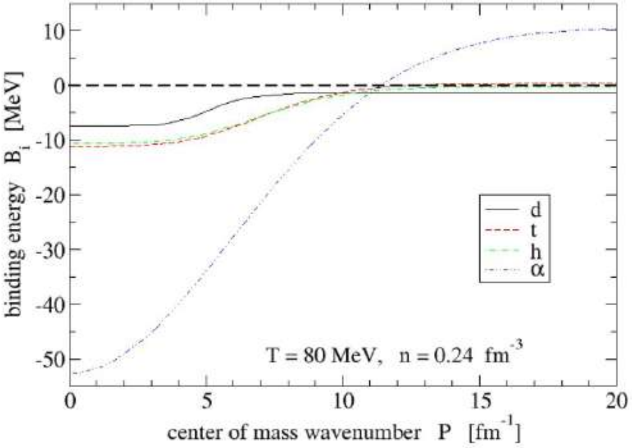
The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

# Momentum-dependent binding energies




momentum space

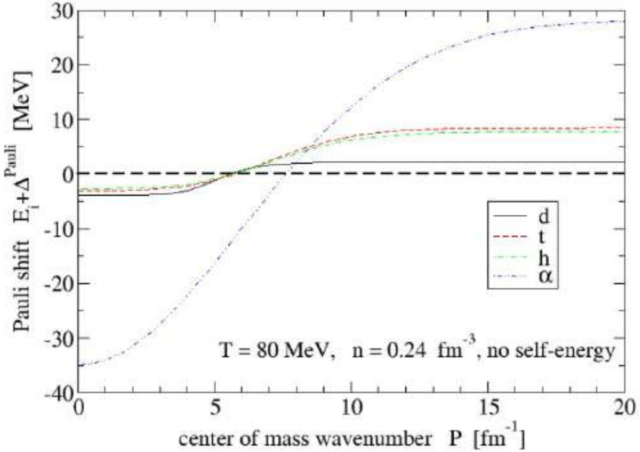



The light clusters that underwent a Mott Dissociation for low momenta become "resurrected" at high momenta relative to the medium !

The minimal momentum where this Occurs is called "Mott momentum"; It depends on temperature and density

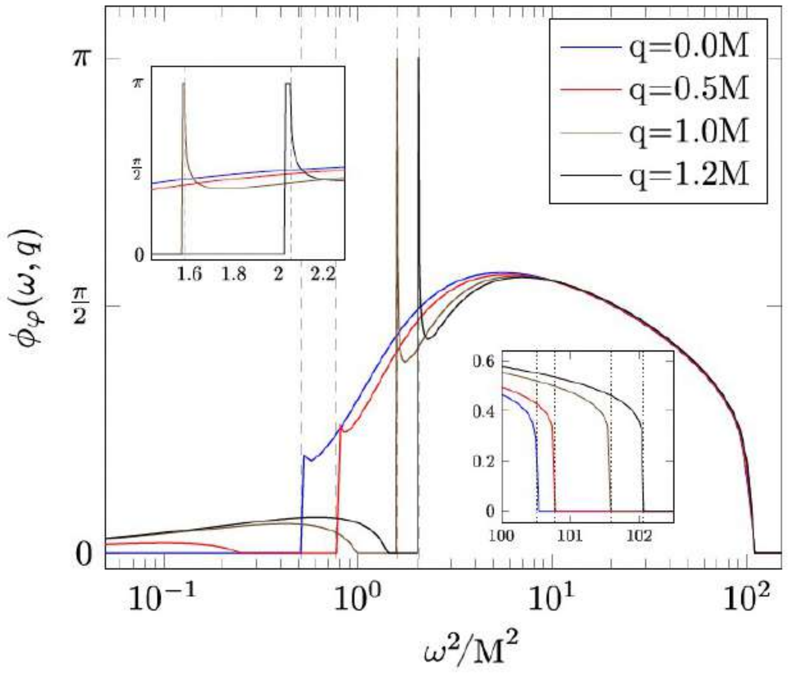
Binding energies without selfenergy shift, Only Pauli blocking shift accounted for

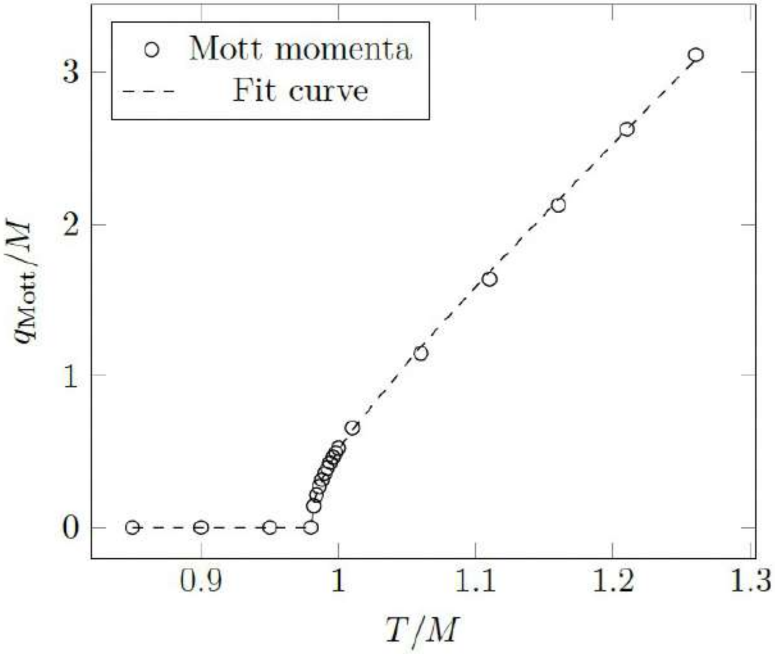


# Mott momentum for bound states in matter

Based on the Pauli principle (100th anniversary), the Mott momentum is a general effect for bound states in matter, e.g. excitons in graphene!

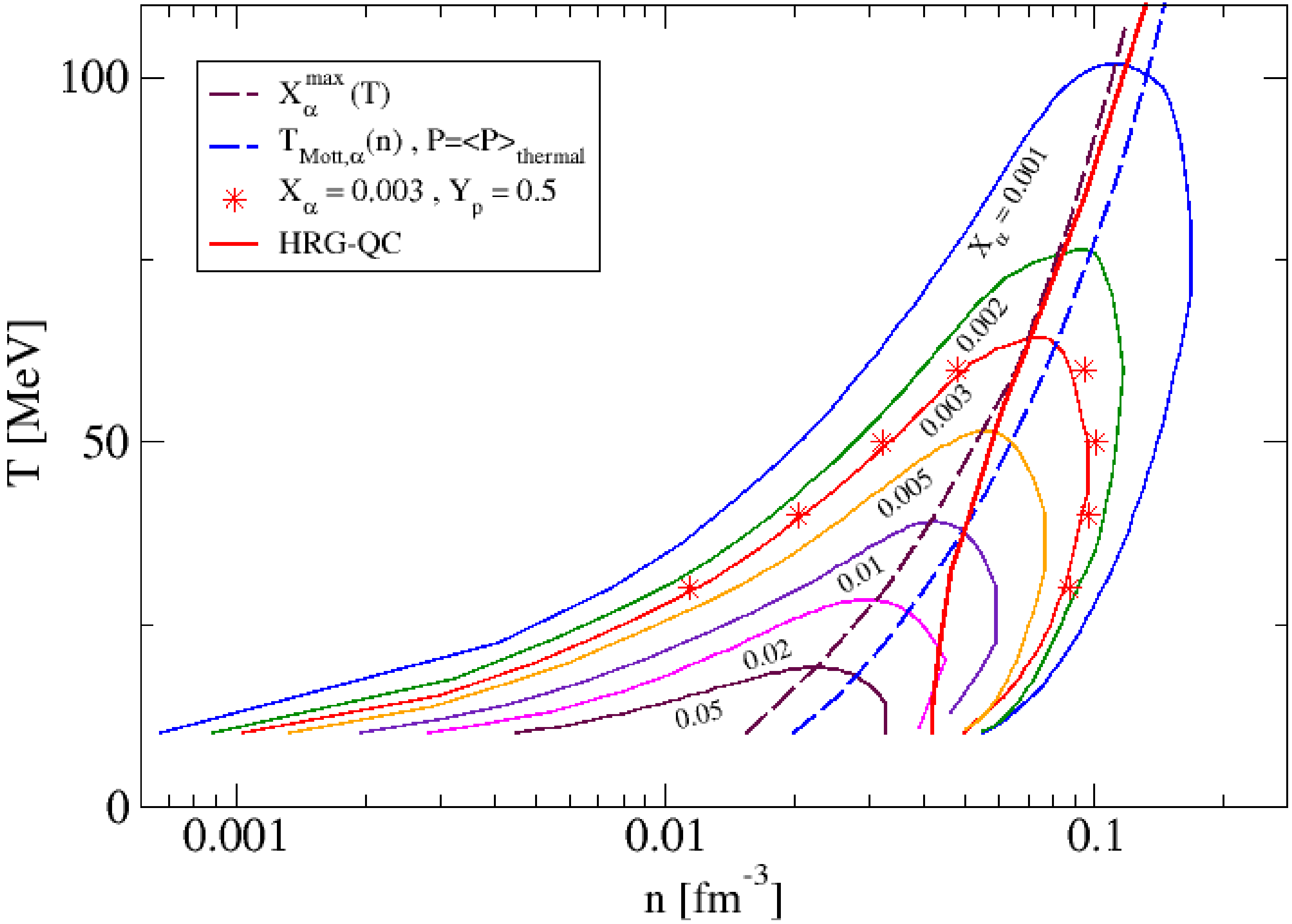
Biplab Mahato, D.B., D. Ebert,
Beth-Uhlenbeck equation for the thermodynamics of fluctuations in a generalized 2+1D Gross-Neveu model (in preparation)




See also: Pauli potential effects on the tetraquark spectrum [Morgan Kuchta, Master Thesis, University of Wroclaw (2024)]

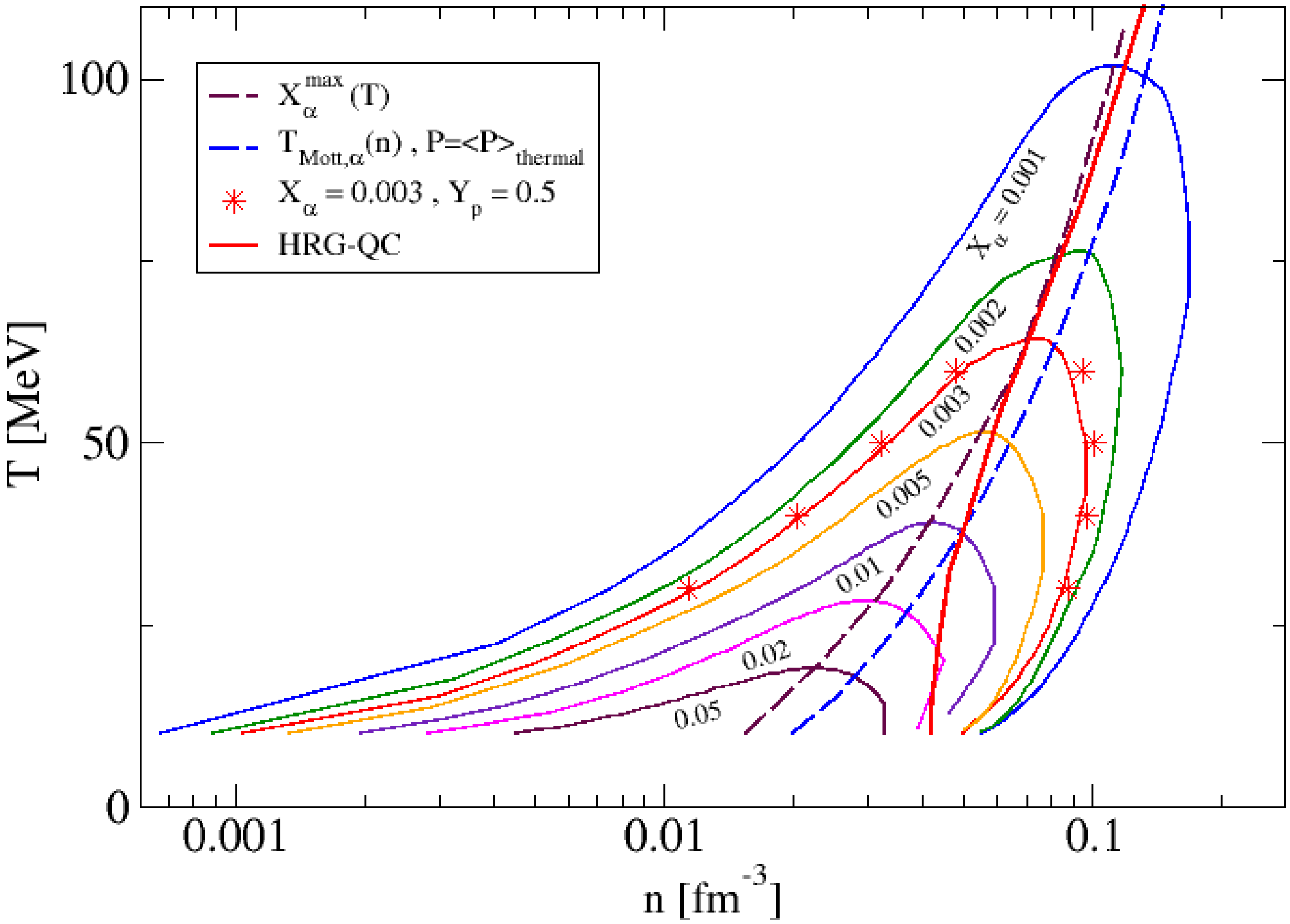
# CFO in the Temperature – density plane




The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

# CFO in the Temperature – density plane



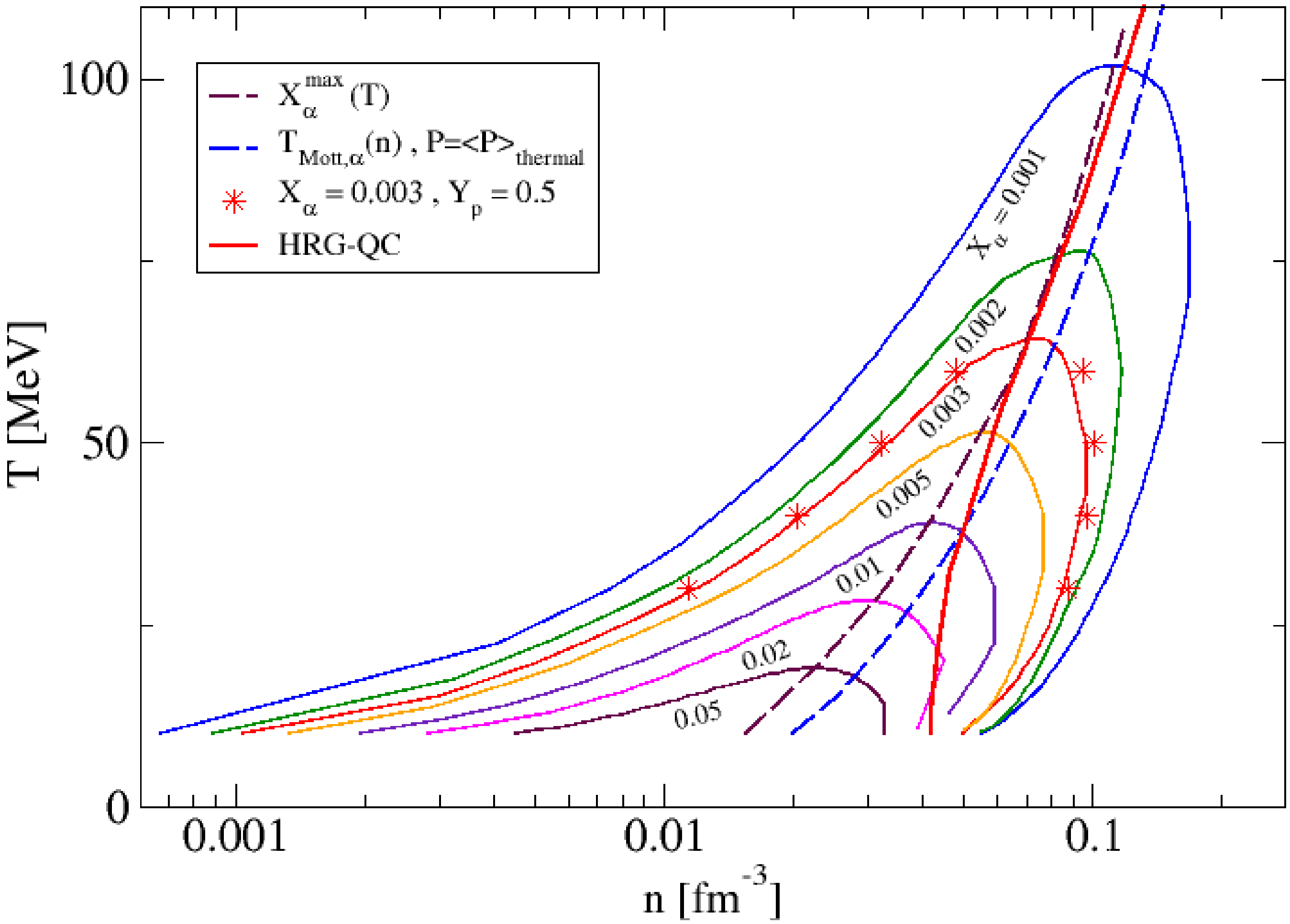

The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

**Pauli blocking →**

**→ Mott dissociation**

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

# CFO in the Temperature – density plane



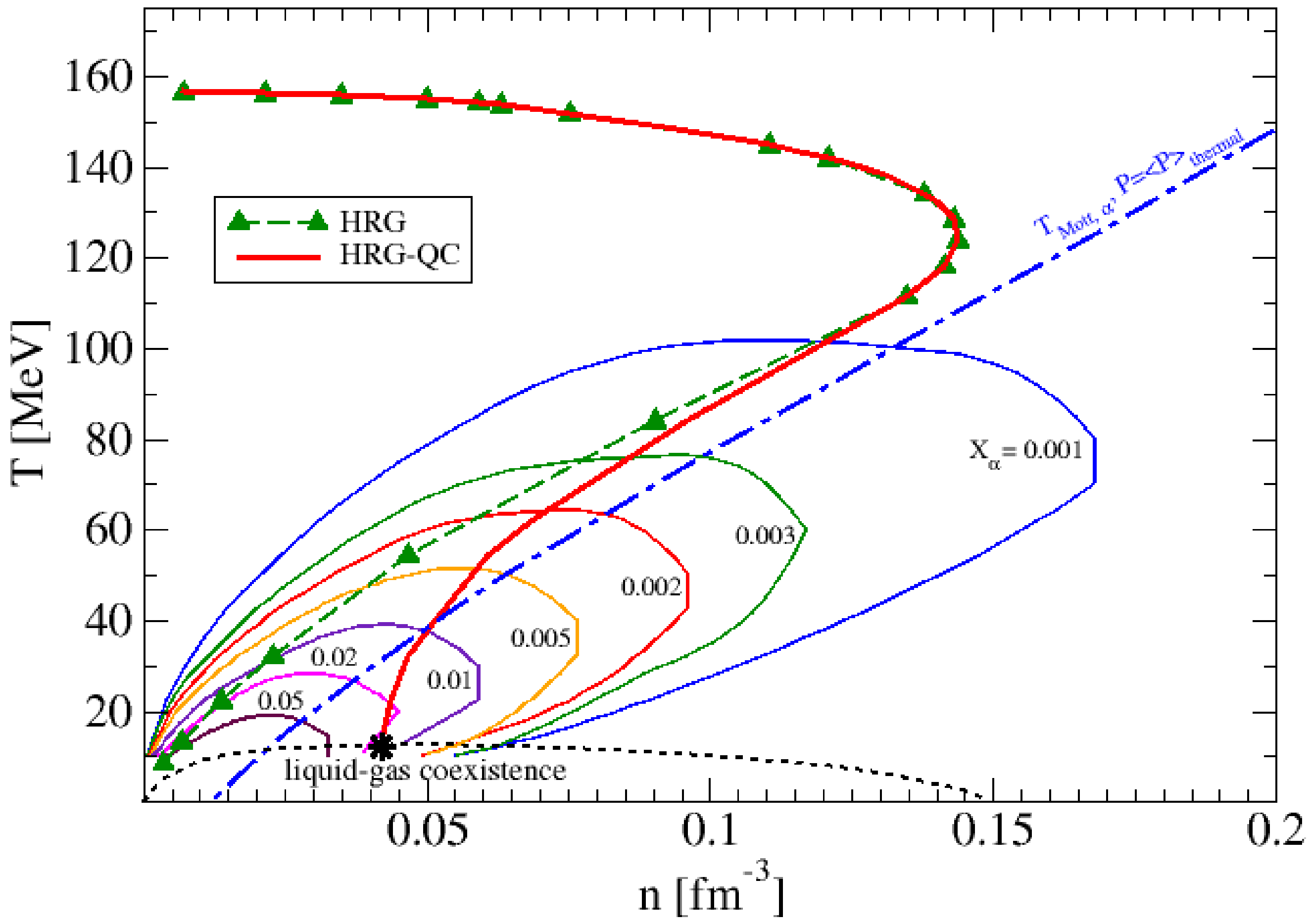

The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

**Pauli blocking →**
**→ Mott dissociation**

Mott-line for alpha clusters (equivalent to the line of maximum alpha fraction) is well correlated with the Chemical Freeze-Out (CFO) line

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

# CFO in the Temperature – density plane




**Main result:**

Chemical freeze-out may be interpreted as „inverse" Mott transition:

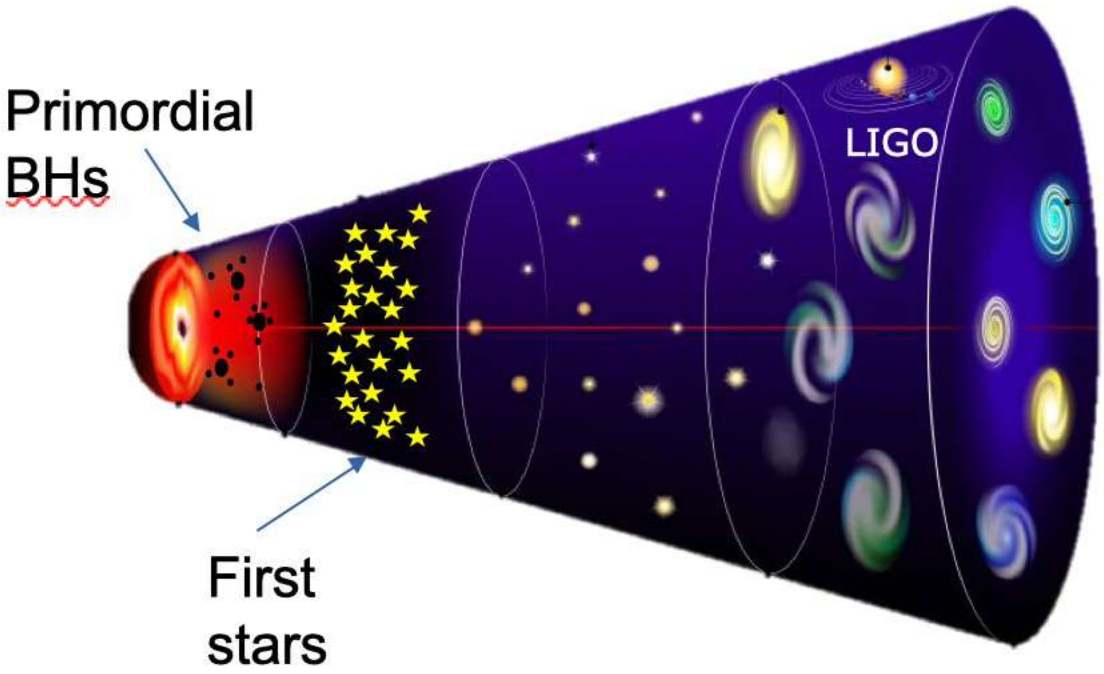
Strong localization effect of nucleon-nucleon correlations in bound states (clusters) entails freeze out of the nuclear composition

„collapse of wave function"

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

# Outlook: Primordial CFO of heavy elements?

## JWST results – primordial black holes !



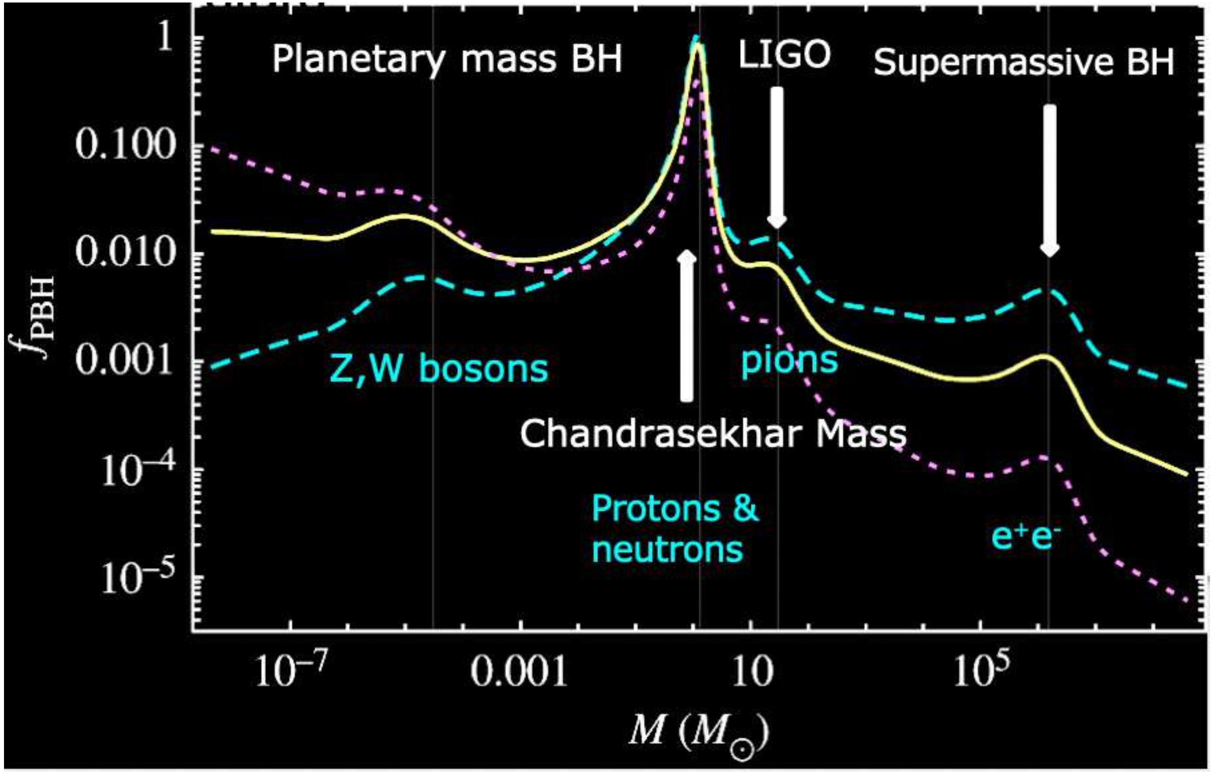

Talk at University of Wroclaw
by Günther Hasinger,
Founding director of the
German Centre for Astrophysics
In Görlitz:

DZA

**QCD hadronization transition plays key role plays for PBH formation !**

# Outlook: Primordial CFO of heavy elements?

## JWST results – primordial black holes !




Different peaks correspond to different particles created at the early universe phase transitions and the corresponding reduction in the sound velocity.

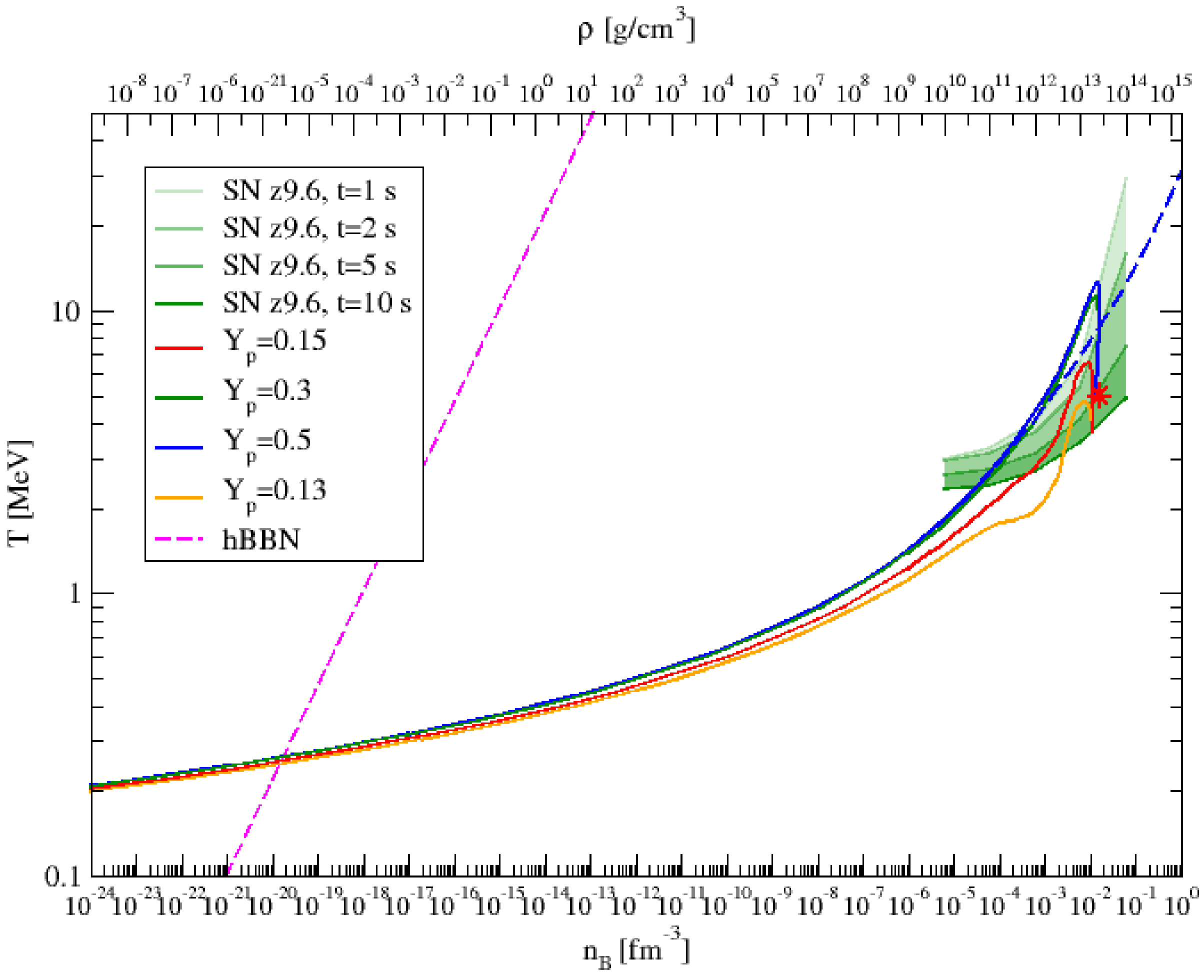
BH mass corresponds to the horizon size at each time.

Only requirement is enough fluctuation power in a volume fraction of $10^{-9}$ of the early Universe.

**Carr, Clesse, García-Bellido 2019**

**QCD hadronization transition plays key role plays for PBH formation !**

# Outlook: Primordial CFO of heavy elements?




Can the primordial evolution of the Universe lead to these freeze-out Parameters (red star):

T=5 MeV,
$\mu_n$=940.317 MeV,
$\mu_p$= 845.069 MeV

Maybe inhomogeneous Big Bang?

The freeze-out point lies in the domain of supernova explosions and binary neutron star mergers

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

# Outlook: Primordial CFO of heavy elements?

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

## Cosmic separation of phases

Edward Witten*
*Institute for Advanced Study, Princeton, New Jersey 08540*
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

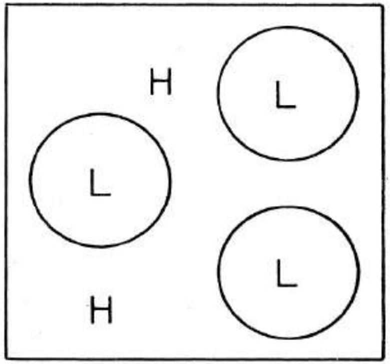



FIG. 1. Isolated expanding bubbles of low-temperature phase in the high-temperature phase.

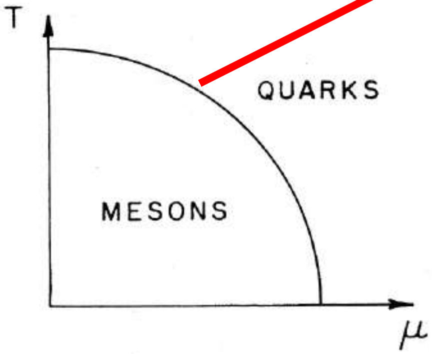
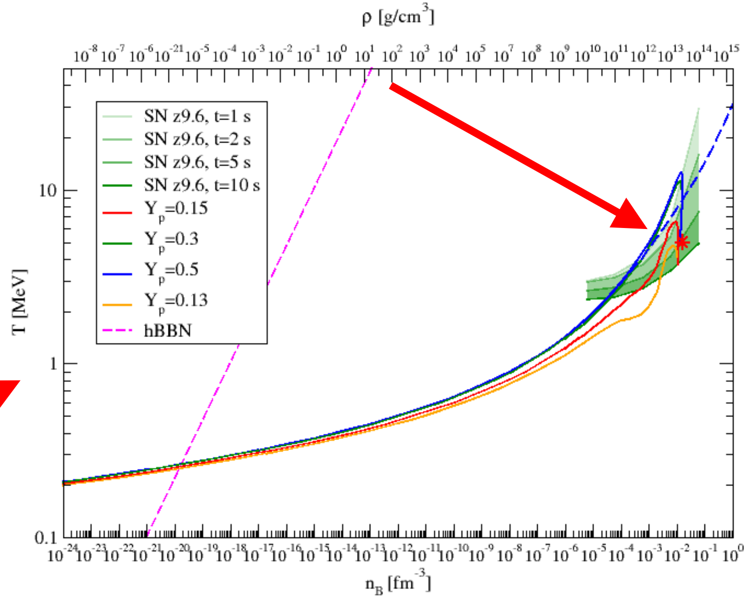


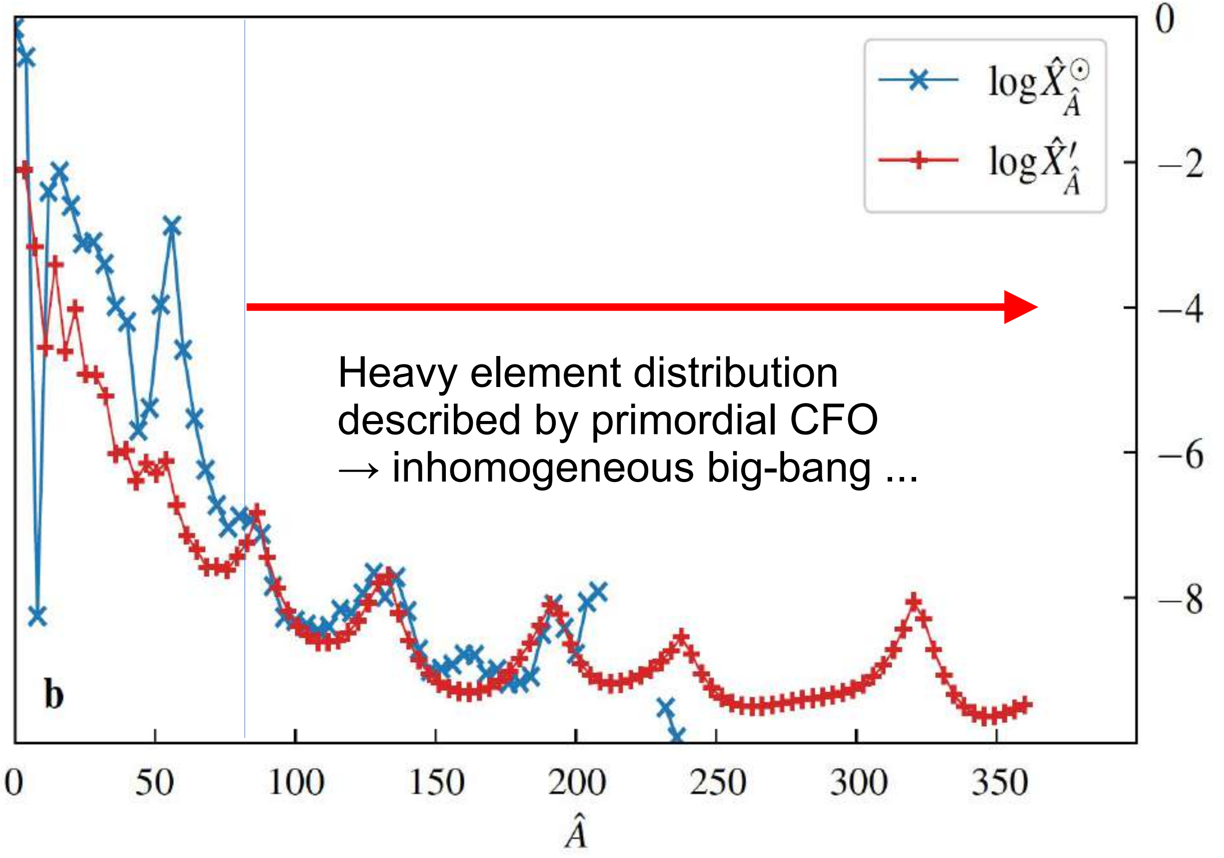

FIG. 4. A sketch of the coexistence temperature for quark matter of chemical potential $\mu$ coexisting with the meson-baryon phase of $\mu=0$. What is shown is the temperature, as a function of $\mu$, at which the two phases exert equal pressure.



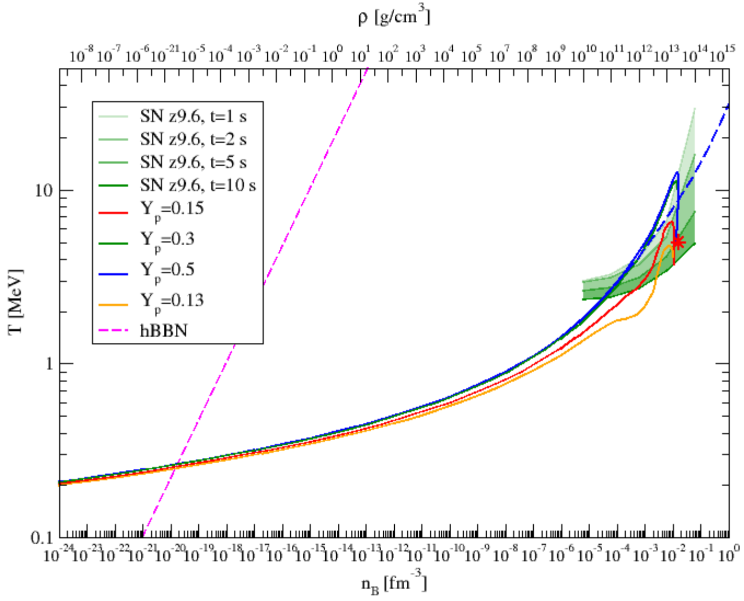

Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, $\mu_n$=940.317 MeV, $\mu_p$= 845.069 MeV

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

# Outlook: Primordial CFO of heavy elements?




Heavy element distribution described by primordial CFO → inhomogeneous big-bang ...




Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, $\mu_n$=940.317 MeV, $\mu_p$= 845.069 MeV

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

# 61st Karpacz Winter School of Theoretical Physics and ChETEC-INFRA Training School – "Multi-messenger nuclear astrophysics in the 21st century"

Uniwersytet Wrocławski

RESEARCH UNIVERSITY EXCELLENCE INITIATIVE

arqus European University Alliance

1-7 March 2025
Artus Hotel
Europe/Warsaw timezone



Contact

tobias.fischer@uwr.edu.pl

ChETEC INFRA

EMMI

HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

IReNA

This 61st edition of the Karpacz Winter School of Theoretical Physics is jointly organised by the Institute of Theoretical Physics and by the Institute of Astronomy, both of the University of Wroclaw (Poland), the Helmholtz-Centre Dresden-Rossendorf (Germany), the Department of Physics and Astronomy of the University of Uppsala (Sweden) and the German Centre for Astrophysics in Görlitz (Germany)

The school aligns with the forthcoming results of the most recent gravitational-wave (GW) observations, specifically from the O4 run of advanced LIGO-VIRGO and the Japanese KAGRA detector, projected for conclusion by the end of 2024. A profound understanding of the growing amount of data necessitates inter-disciplinary expertise in astrophysics and astronomy, numerical relativity, as well as nuclear and particle physics experiment and theory, which is essential for grasping the physics implications of multi-messenger observations associated with the GW detections. With the focus on compact object mergers and collapsing stars as well as their nuclear astrophysics implications, this school endeavours to enhance and stimulate the information flow and close collaboration between researchers from these different disciplines.

## Subjects:

- Explosive astrophysical phenomena – the physics of supernovae and neutron star mergers
- Astrophysical messengers – neutrinos, gravitational waves and gamma-rays
- Nuclear astrophysics experiments and theory
- Nucleosynthesis and galactic chemical evolution
- Metal poor stars and their observations

https://events.ift.uni.wroc.pl/event/68/

# Backup Slides

# Mott-Anderson localization model for sudden CFO

**The basic idea: Localization of (certain) multiquark states ("cluster") = hadronization;**
**Reverse process = delocalization by quark exchange between hadrons**

**Freeze-out criterion:**

$$H_{\exp}(\tau) = \frac{\dot{R}(\tau)}{R(\tau)} = \tau^{-1}_{\mathrm{coll},i}(T,\mu) \ ,$$

$$\tau^{-1}_{\mathrm{coll},i}(T,\mu) = \sum_j \sigma_{ij} v n_j(T,\mu)$$

**Povh-Huefner law, PRC 46 (1992) 990 → total x-section**

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

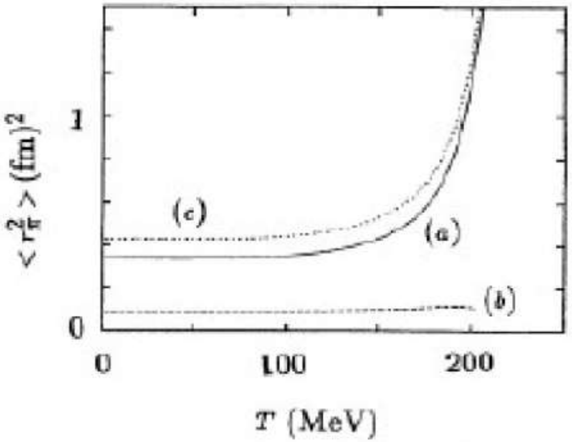
$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu)$$

**Hippe & Klevansky, PRC 52 (1995) 2172**

$$f_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / M_\pi^2$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} \left| \langle \bar{q}q \rangle_{T,\mu} \right|^{-1}$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\mathrm{MF}} \left[ 1 - \frac{T^2}{8 f_\pi^2(T,\mu)} - \frac{\sigma_N n_{s,N}(T,\mu)}{M_\pi^2 f_\pi^2(T,\mu)} \right]$$




DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

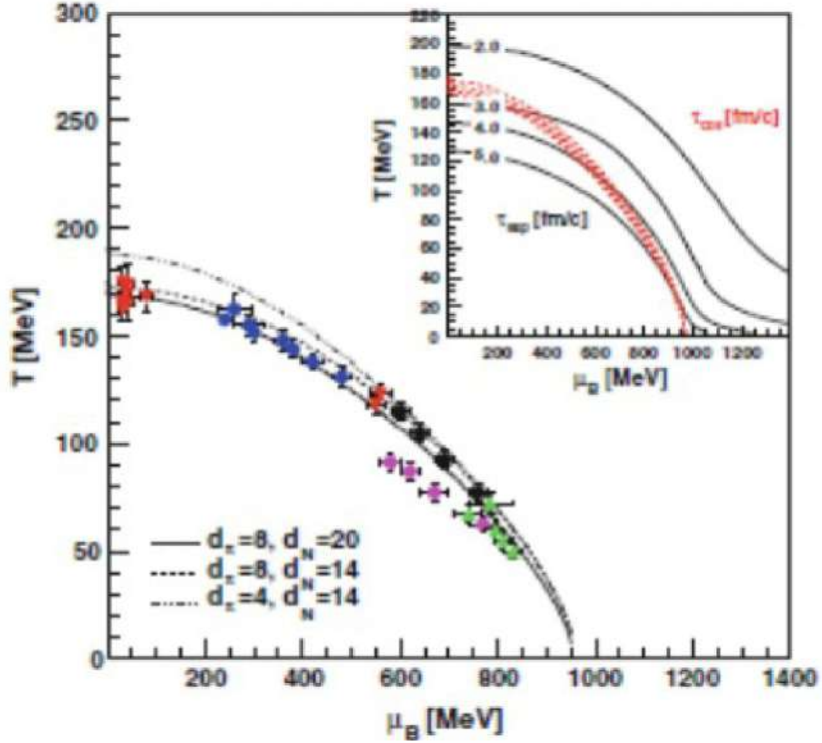
# Mott-Anderson localization model for sudden CFO

**Model results:**

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

**Collision time strongly T, mu dependent !**

**Schematic resonance gas: dπ pions, dN nucleons**

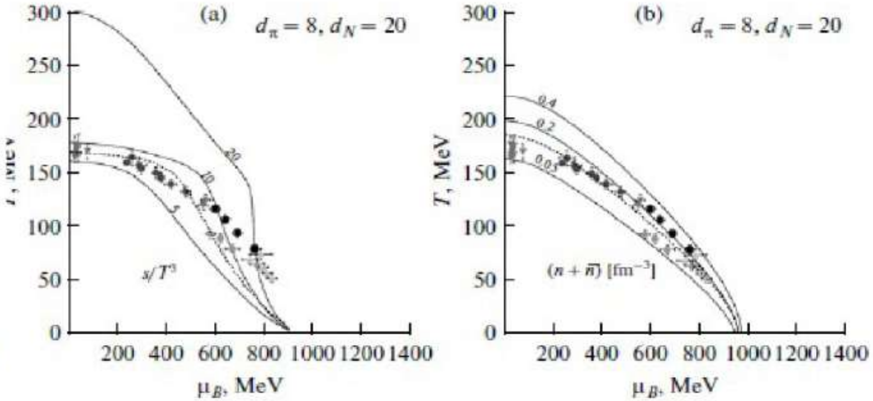



**Expansion time scale from entropy conservation:**

$$s(T, \mu)\, V(\tau_{exp}) = \text{const}$$

$$\tau_{exp}(T, \mu) = a s^{-1/3}(T, \mu),$$

**Thermodynamics consistent with phenomenological Freeze-out rules:**




DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

# Mott-Anderson localization model - refined

A) Chiral condensate for the full hadron resonance gas model → radii of hadrons

- nonstrange hadrons:

$$\langle r_\pi^2 \rangle_{T,\mu} = \frac{3}{4\pi^2 f_\pi^2} \qquad f_\pi^2(T,\mu) = \frac{-m_q \langle \bar{q}q \rangle_{T,\mu}}{m_\pi^2} ,$$

$$\langle r_\pi^2 \rangle_{T,\mu} = \frac{3 m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1} \qquad \langle r_{\rm N}^2 \rangle_{T,\mu} = r_0^2 + \langle r_\pi^2 \rangle_{T,\mu}$$

- strange hadrons:

$$f_K^2 m_K^2 = -\frac{\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}}{2} (m_q + m_s)$$

$$\langle r_{\rm K}^2 \rangle_{T,\mu} = \frac{3}{4\pi^2 f_{\rm K}^2} = \frac{3}{2\pi^2} \frac{m_{\rm K}^2}{|\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}|(m_q + m_s)} \qquad \langle r_\Lambda^2 \rangle_{T,\mu} = r_0^2 + \langle r_{\rm K}^2 \rangle_{T,\mu}$$

B) Chemical freeze-out: only "reactive" cross section, flavor equilibration

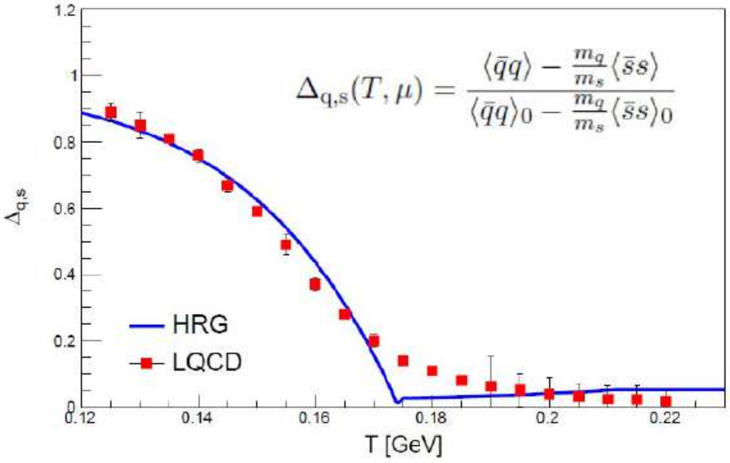
Some flavor changing processes involve reaction thresholds and need activation energy, like in the Eyring theory of chemical processes with activation:

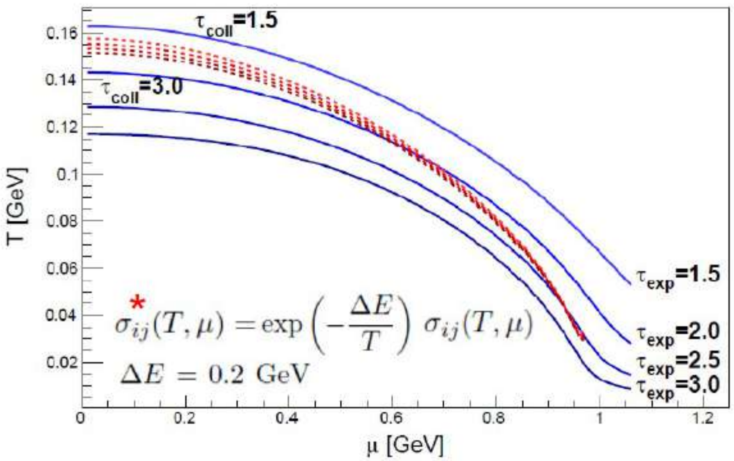
$$\sigma_{ij}^{*}(T,\mu) = \exp\left(-\frac{\Delta E}{T}\right) \sigma_{ij}(T,\mu) \qquad \sigma_{ij}(T,\mu) = \lambda \langle r_i^2 \rangle_{T,\mu} \langle r_j^2 \rangle_{T,\mu}$$

Assumption: average activation threshold for reactive processes: $\Delta E = 0.2$ GeV
(to be refined, account for all individual processes, e.g., SMASH)

DB, J. Jankowski, M. Naskret, arxiv:1705.00169

# Mott-Anderson localization model - refined




$$\langle \bar{q}q\rangle_{T,\mu} = \langle \bar{q}q\rangle^{MF}_{T,\mu} + \sum_{h=M,B} \frac{\sigma^h_q}{m_q} n_h(T,\mu) \ ,$$

$$n_h(T,\mu) = \frac{d_h}{2\pi^2}\int_0^\infty dk k^2 \frac{m_h}{E_h} \frac{1}{e^{(E_h-\mu_h)/T} \mp 1} \ .$$

$$\tau^{-1}_{\mathrm{coll},i}(T,\mu) = \sum_j \sigma^*_{ij} v n_j(T,\mu) \ ; \ \sigma_{ij} = \lambda \langle r_i^2\rangle \langle r_j^2\rangle$$

$$\langle r_\pi^2\rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} \left|\langle \bar{q}q\rangle_{T,\mu}\right|^{-1}$$

$$\langle r_K^2\rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2(m_q+m_s)} \left|\langle \bar{q}q\rangle_{T,\mu} + \langle \bar{s}s\rangle_{T,\mu}\right|^{-1}$$

The factor a stands for the inverse system size in the formula

$$\tau_{\mathrm{exp}}(T,\mu) = \tau_{\mathrm{coll}}(T,\mu)$$

for the 3D expansion time scale assuming entropy conservation

Full HRG model condensate;
J. Jankowski et al., Phys. Rev. D (2013)

DB, J. Jankowski, M. Naskret, arxiv:1705.00169

# Mott-Anderson localization model - refined

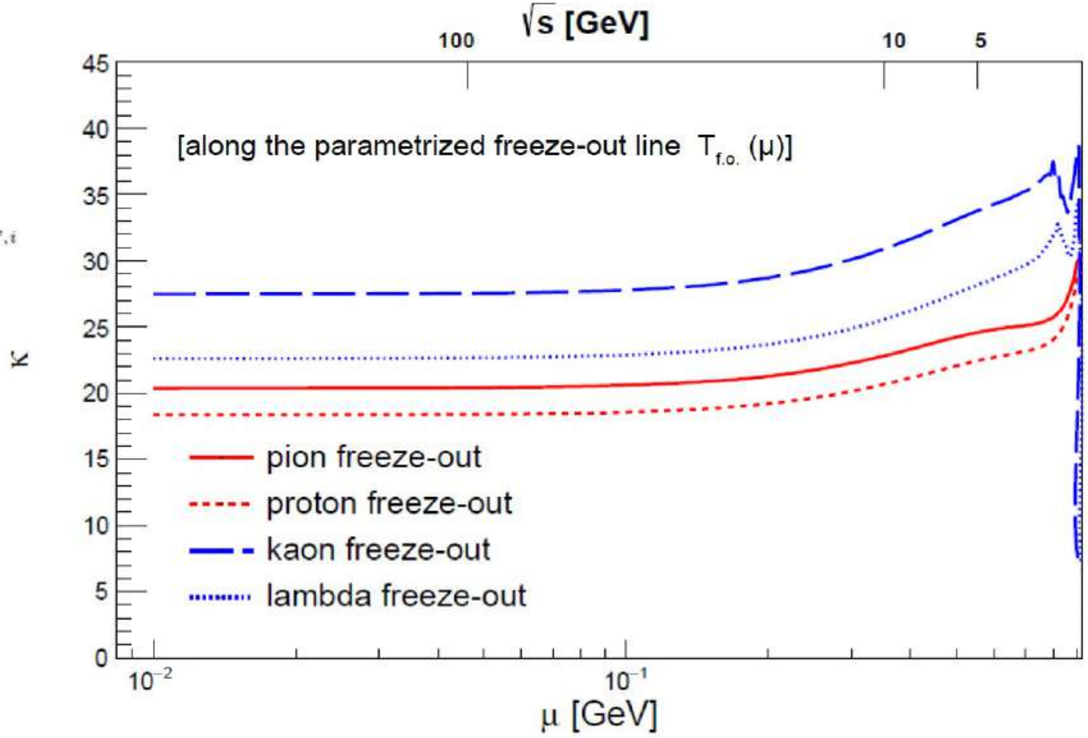
Inelastic collision rate $\tau_{\rm coll} \propto T^{\kappa}, \quad \kappa \gtrsim 20$ from fit to STAR data

U. Heinz and G. Kestin, PoS CPOD **2006**, 038 (2006) [nucl-th/0612105]

Species-dependent exponent of the power law,

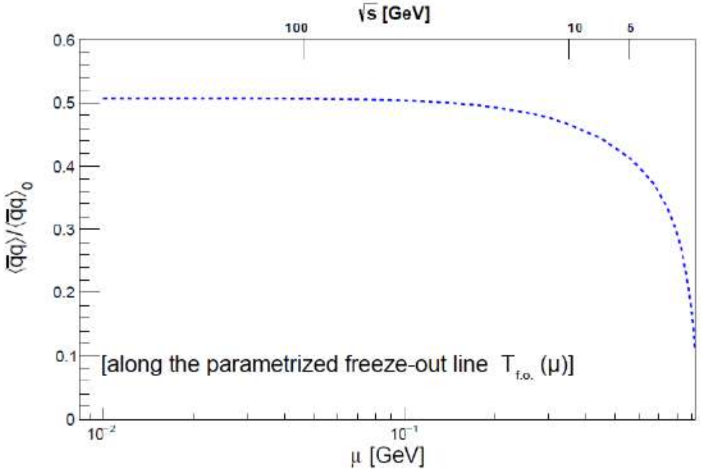
$$\kappa_i = -\frac{d\ln\tau_{{\rm coll},i}(T,\mu)}{d\ln T}\Bigg|_{T_{f,i};\mu_{f,i}}$$

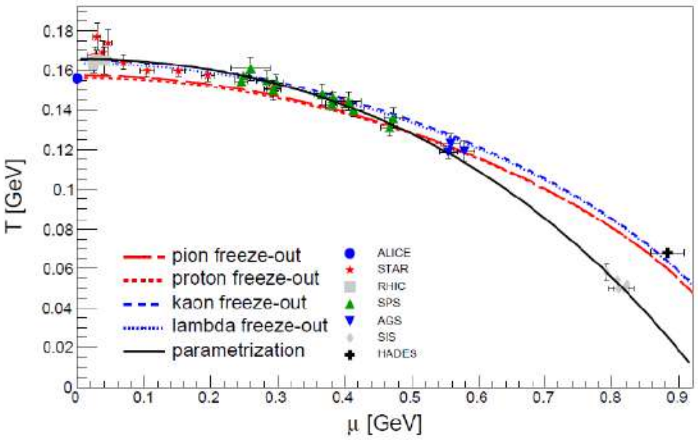
extracted from the model for the collision rate.




DB, J. Jankowski, M. Naskret, arxiv:1705.00169

# Mott-Anderson localization model - refined




$$\langle\bar{q}q\rangle_{T,\mu} = \langle\bar{q}q\rangle^{MF}_{T,\mu} + \sum_{h=M,B} \frac{\sigma_q^h}{m_q} n_h(T,\mu)\ ,$$

$$n_h(T,\mu) = \frac{d_h}{2\pi^2}\int_0^\infty dk k^2 \frac{m_h}{E_h}\frac{1}{e^{(E_h-\mu_h)/T}\mp 1}\ .$$

$$\tau^{-1}_{\mathrm{coll},i}(T,\mu) = \sum_j \sigma^{*}_{ij} v n_j(T,\mu)\ ;\quad \sigma_{ij} = \lambda\,\langle r_i^2\rangle\,\langle r_j^2\rangle$$

$$\langle r_\pi^2\rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar{q}q\rangle_{T,\mu}\right|^{-1}$$

$$\langle r_K^2\rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2(m_q+m_s)}\left|\langle\bar{q}q\rangle_{T,\mu}+\langle\bar{s}s\rangle_{T,\mu}\right|^{-1}$$

**The factor a stands for the inverse system size in the formula**

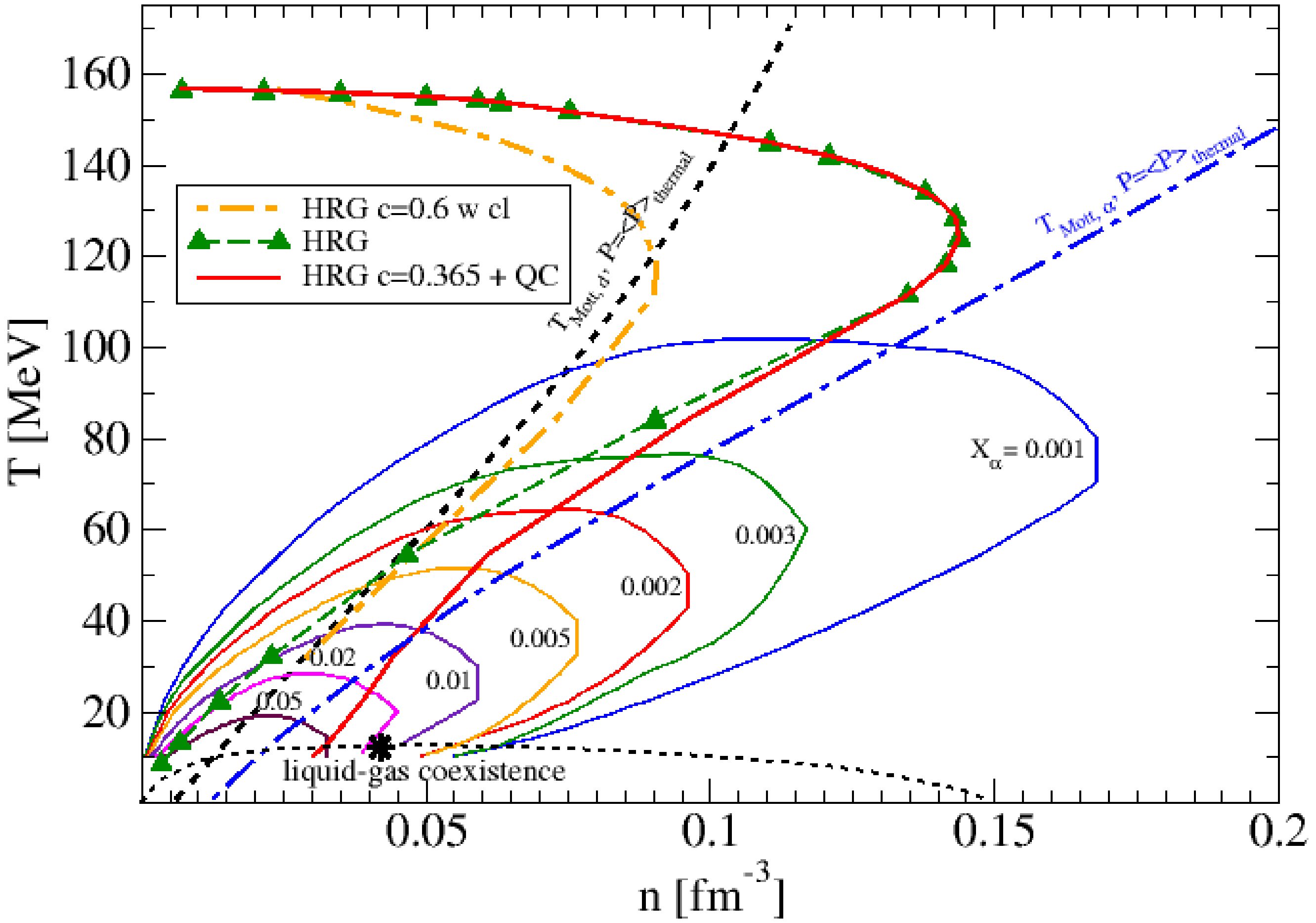
$$\tau_{\mathrm{exp}}(T,\mu) = \tau_{\mathrm{coll}}(T,\mu)$$

**for the 3D expansion time scale assuming entropy conservation**

Full HRG model condensate;
J. Jankowski et al., Phys. Rev. D (2013)

DB, J. Jankowski, M. Naskret, arxiv:1705.00169

# CFO in the Temperature – density plane



# CFO in the Temperature – density plane