Pseudoconformal behavior of color-flavor-locked quark matter within a nonlocal chiral quark model

Oleksii Ivanytskyi

OI, PRD 111, 034004 (2025) + update

Wroclaw, 28 April 2025

Phase diagram of strongly interacting matter

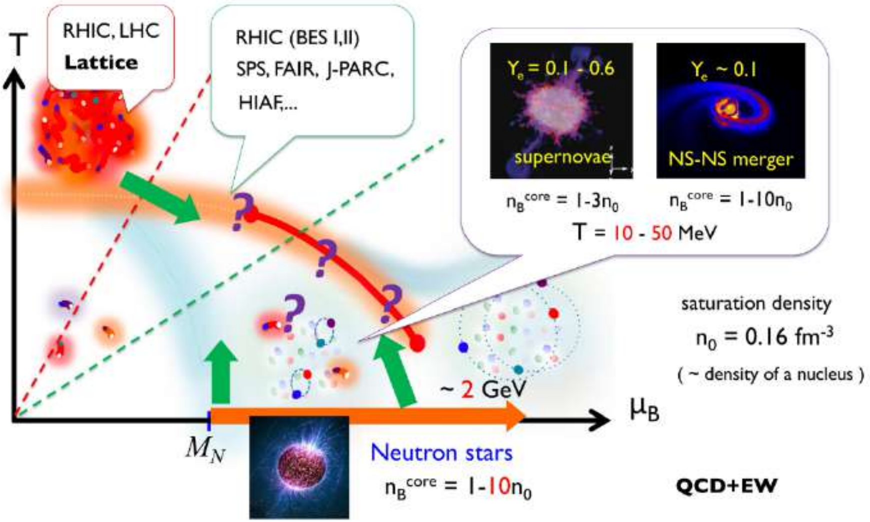
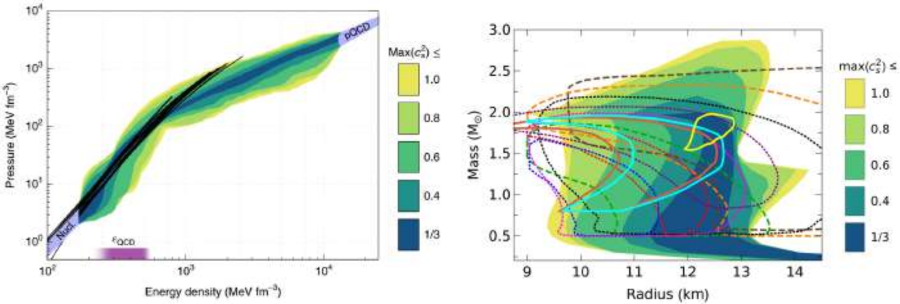


Figure from T. Kojo arXiv:1912.05326 [nucl-th]

Quark matter in neutrons stars? pQCD vs $2M_\odot$



E. Annala, T. Gorda, A. Kurkela, J. Nättilä, A. Vuorinen, Nature Physics 16, 907 (2020)

Existence of parameterization consistent with pQCD and $2M_\odot$

$\Downarrow$

Argument in favor of quark cores?

Quark matter in neutrons stars? Supernova explosions

- $2 M_\odot$ stars formation? (accretion is too slow)
- Supernovae with progenitor mass $\sim 50\ M_\odot$
- Quark-hadron transition stabilizes collapse

T. Fischer et al., Nature Astronomy 2, 980–986 (2018)

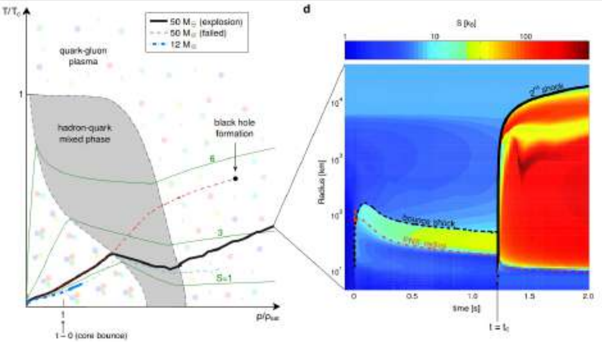


Table 1 | Summary of the supernova simulation results with hadron–quark phase transition

M_{ZAMS} ($M_\odot$)	t_{onset} (s)	$t_{collapse}$ (s)	$\rho\vert_{collapse}$ (ρ_{sat})	$T\vert_{collapse}$ (MeV)	$M_{PNS,collapse}$[a] ($M_\odot$)	t_{final} (s)	$\rho\vert_{final}$ (ρ_{sat})	$T\vert_{final}$ (MeV)	$M_{PNS,final}$[a] ($M_\odot$)	E^*_{expl} (10^{51} erg)
12[12]	3.251	3.489	2.49	28	1.727	3.598	5.5	17	1.732	0.1
18[12]	1.465	1.518	2.53	27	1.958	1.575	5.9	18	1.964	1.6
25[14]	0.905	0.976	2.40	31	2.163	0.983	9.6	19	2.171[b]	-
50[15]	1.110	1.215	2.37	32	2.105	1.224	5.8	31	2.092	2.3

Deconfinement is a supernova engine for massive blue giants

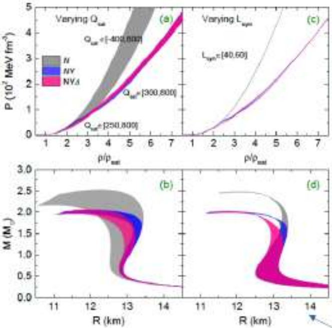


FIG. 4. EoS models and MR relations for N, NY, and $NY\Delta$ compositions of stellar matter. The bands are generated by varying the parameters Q_{sat} [MeV] (a, b) and L_{sym} [MeV] (c, d). The ranges of Q_{sat} and L_{sym} allowed by χEFT and maximum mass constraints are indicated in the figures.

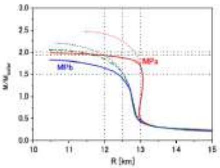


FIG. 7. Neutron-star masses as a function of the radius R. Solid (dashed) curves are with (without) hyperon (Λ and Σ^-) mixing for ESC+MPa and ESC+MPb. The dot-dashed curve for MPb is with Λ mixing only. Also see the caption of Fig. 3.

Yamamoto et al., Phys.Rev.C 96 (2017) 06580; arXiv:1708.06163 [nucl-th]
Yamamoto et al., Eur. Phys. J. A 52 (2016) 19; arXiv:1510.06099 [nucl-th]
Ji & Sedrakian, Phys. Rev. C 100 (2019) 015809; arXiv:1903.08057 [astro-ph.HE]

Examples for realistic hadronic EoS which suggest a Berlin Wall is inferior to the line M = 2.0 M_sun

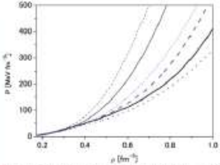


Fig. 8. Pressure P as a function of baryon density ρ. Thick (thin) curves are with (without) hyperon mixing. Solid, dashed and dotted curves are for MPa, MPa$^+$ and MPb.

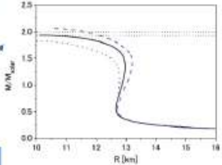


Fig. 9. Neutron-star masses as a function of the radius R. Solid, dashed and dotted curves are for MPa, MPa$^+$ and MPb. Two dotted lines show the observed mass $(1.97 \pm 0.04)M_\odot$ of J1614-2230.

Hyperons soften EoS

$\Downarrow$

EoS can be stiffened by quarks

Conformal symmetry vs equation of state (EoS)

- Generators of the conformal group

$$
\begin{aligned}
M_{\mu\nu} &= i(x_\mu \partial_\nu - x_\nu \partial_\nu) && - \textbf{boosts/rotations} \\
P_\mu &= -i\partial_\mu && - \textbf{translations} \\
D &= -ix_\mu \partial^\mu && - \textbf{rescaling} \\
K_\mu &= i(x^2 \partial_\mu - 2x_\mu x_\nu \partial^\nu) && - \textbf{special transformation}
\end{aligned}
$$

- Scale invariance + thermodynamic identities

$$[p] = [\text{energy}^{d+1}]$$

$$[\mu] = [\text{energy}] \qquad \Rightarrow \qquad \varepsilon = dp, \quad c_S^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{d}$$

$$\varepsilon = \mu \frac{\partial p}{\partial \mu} - p$$

QCD @ high densities

- Asymptotic freedom
- Negligible quark mass

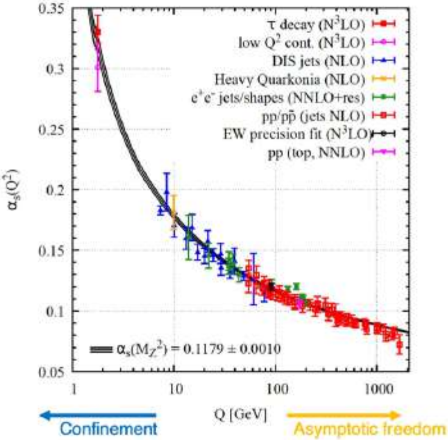
$$m \ll \mu$$

- EoS

$$p = \frac{N_c N_f \mu^4}{12\pi^2} + corrections$$

$$\delta \equiv \frac{T_{\mu\mu}}{3T_{00}} = \frac{1}{3} - \frac{p}{\varepsilon} \rightarrow 0$$

$$c_S^2 \equiv \frac{\partial p}{\partial \varepsilon} \rightarrow \frac{1}{3}$$



taken form ATLAS Open Data

Is conformal behavior a signature of quark matter in NS?

Conformality vs quark matter in neutron stars

- **Assumption:**

  quark matter can be approximately conformal already @ the neutron star densities

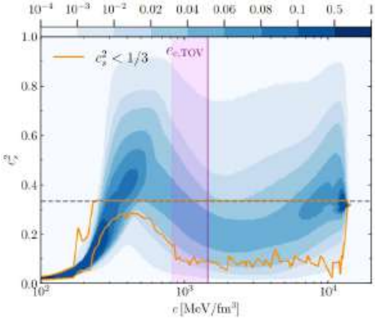
  Caution: non-conformal behavior does not mean absence of quark matter

- **Criteria of approximate conformal behavior**
  - $K_{NM} = 9n_B^2 \frac{\partial^2}{\partial n_B^2} \frac{\varepsilon}{n_B} \simeq -\frac{3\mu_B}{2}$

    OI, D. Blaschke, Particles 5, 4, 514 (2022)
  - $d_c = \sqrt{\delta^2 + \left(\varepsilon \frac{d\delta}{d\varepsilon}\right)^2} < d_c^{hadron} \simeq 0.2$

    E. Annala et al., Nature Commun. 14, 845 (2023)
  - $\beta_c \equiv \frac{K_{NM}}{9\mu_B} = c_S^2 - \frac{2-6\delta}{3-3\delta} < 0$

    M. Marczenko et al. PRD 109, L041302 (2024)
  - ...



S. Altiparmak et al., Astrophys.J.Lett. 939, 2, L34 (2022)

None of the criteria is microscopic. How reliable they are?

Does their breaking mean absence of quark matter?

Model to test conformality in neutron stars

- Quark degrees of freedom
- Asymptotically conformal behavior in agreement with pQCD
- Consistency with observational constraints on properties of neutron stars

$N_f = 3$ nonlocal NJL model with vector repulsion and diquark pairing

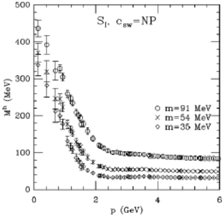
Nonlocal character of quark interactions

- QCD interactions

- Indications from the lattice

p-dependent renormalization of quark propagator
(p-dependent quark mass)
$\Downarrow$
space-time nonlocality of quark interactions

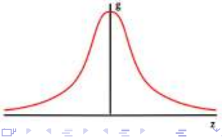
J. Skullerud, D. B. Leinweber, and A. G. Williams, Phys. Rev. D 64 , 074508 (2001)



- Quark interactions in effective approaches (separable approximation)

$$\left(\overline{q}_x \hat{\Gamma} q_x\right)^2 \rightarrow \left(\int dz\ \tilde{g}_z\ \overline{q}_{x+z/2} \hat{\Gamma} q_{x-z/2}\right)^2$$

$\tilde{g}_z$ – space-time dependent fromfactor



Nonlocal NJL model for three-flavor quark matter

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m + \mu\gamma_0)q + G_S \sum_{a=\overline{0,8}} s_a s_a - G_V j_\mu j^\mu + 3G_D \sum_{a,b=2,5,7} d^+_{ab} d_{ab}$$

- Scalar interaction channel – chiral symmetry breaking/restoration

$$s_a(x) = \int dz\ \tilde{g}(z)\overline{q}\left(x + \frac{z}{2}\right) \tau_a q \left(x - \frac{z}{2}\right)$$

  τ_a - generators of SU(3) flavor group
- Vector interaction channel – repulsion

$$j_\mu(x) = \int dz\ \tilde{g}(z)\overline{q}\left(x + \frac{z}{2}\right) \gamma_\mu q \left(x - \frac{z}{2}\right)$$

- Diquark pairing interaction channel – CFL color superconductivity

$$d_{ab}(x) = \int dz\ \tilde{g}(z)\overline{q}\left(x + \frac{z}{2}\right) i\gamma_5\tau_a\lambda_b q^c \left(x - \frac{z}{2}\right)$$

  λ_b - generators of SU(3) color group
- Gaussian formfactor in instantaneous approximation

$$g(k) \equiv \int dz\ e^{ikz}\ \tilde{g}(z) = \exp(-\mathbf{k}^2/\Lambda^2)$$

Bozonization & mean-field equations

$$\mathcal{L}^{\rm bos} = \bar{q}(i\partial\!\!\!/ - m + \mu\gamma_0)q - \sum_{a=\overline{0,8}}\left(s_a\sigma_a + \frac{\sigma_a\sigma_a}{4G_S}\right) + j_\mu\omega^\mu + \frac{\omega_\mu\omega^\mu}{4G_V} - \sum_{a,b=2,5,7}\left(\frac{d^+_{ab}\Delta_{ab} + \Delta^*_{ab}d_{ab}}{2} + \frac{\Delta^*_{ab}\Delta_{ab}}{12G_D}\right)$$

m - the same for all quark flavors

- Euler–Lagrange equations @ mean-field approximation

$$\sigma_a = -2G_S s_a \rightarrow -2G_S\langle s_0\rangle\delta_{a0} \equiv \sigma\delta_{a0}$$
$$\omega_\mu = -2G_V j_\mu \rightarrow -2G_V\langle j_0\rangle\delta_{\mu 0} \equiv \omega\delta_{\mu 0}$$
$$\Delta_{ab} = -6G_D d_{ab} \rightarrow -2G_D\sum_{c=2,5,7}\langle d_{cc}\rangle\delta_{ab} \equiv \Delta\delta_{ab}$$

- Mean-field Lagrangian

$$\mathcal{L}^{\rm MF} = \bar{Q}\hat{\mathcal{S}}Q - \frac{\sigma^2}{4G_S} + \frac{\omega^2}{4G_V} - \frac{|\Delta|^2}{4G_D}, \quad \hat{\mathcal{S}}^{-1} = \begin{pmatrix} \hat{S}_+^{-1} & i\Delta g_k\gamma_5\hat{\mathcal{O}} \\ i\Delta^* g_k\gamma_5\hat{\mathcal{O}} & S_-^{-1}\end{pmatrix}$$

$$\hat{S}_\pm^{-1} = k\!\!\!/ - M_k \pm \gamma_0(\mu + \omega g_k),\ M_k = m + g_k\sigma,\ \hat{\mathcal{O}} = \tau_2\lambda_2 + \tau_5\lambda_5 + \tau_7\lambda_7$$

Thermodynamic potential @ $T = 0$

$$
\begin{aligned}
\Omega &= -\frac{1}{\beta V} \ln \int [D\overline{Q}][DQ] \exp\left(\int dx\ \mathcal{L}^{\mathrm{MF}} \right) \\
&= -\sum_{j,a=\pm} d_j \int_{\mathbf{k}} \left[\frac{1}{2} - f^a_{j\mathbf{k}} \right] \epsilon^a_{j\mathbf{k}} + \frac{\sigma^2}{4G_S} - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D}
\end{aligned}
$$

- Single-particle energies

$$
\det \hat{\mathcal{S}}^{-1} = 0 \quad \Rightarrow \quad \epsilon^a_{j\mathbf{k}} = \mathrm{sign}\,(\epsilon^a_{\mathbf{k}}) \sqrt{\epsilon^{a\,2}_{\mathbf{k}} + \Delta^2_{j\mathbf{k}}}, \quad \epsilon^{\pm}_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + M^2_{\mathbf{k}}} \mp \mu \mp \omega g_{\mathbf{k}}
$$

- Mean-field equations

$$
\begin{aligned}
\sigma &= 2G_S \sum_{j,a} d_j \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{1}{2} - f^a_{j\mathbf{k}} \right] \frac{\epsilon^a_{\mathbf{k}}}{\epsilon^a_{j\mathbf{k}}} \frac{M_{\mathbf{k}} g_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} \\
\omega &= 2G_V \sum_{j,a} d^a_j \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{1}{2} - f^a_{j\mathbf{k}} \right] \frac{\epsilon^a_{\mathbf{k}}}{\epsilon^a_{j\mathbf{k}}} g_{\mathbf{k}} \\
\Delta &= 2G_D \sum_{j,a} d_j \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{1}{2} - f^a_{j\mathbf{k}} \right] \frac{\Delta}{\epsilon^a_{j\mathbf{k}}} \zeta^2_j g^2_{\mathbf{k}}
\end{aligned}
$$

Scenario of onset of quark matter in neutron stars

BEC of stable double strange sexaquarks

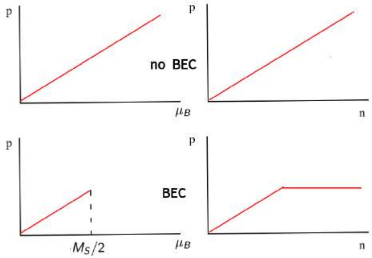
M. Shahrbaf, et al., Phys. Rev. D 105, 103005 (2022)

$\Downarrow$

vanishing incompressibility

$\Downarrow$

gravitational compression induced pile up of density



BEC of sexaquarks $\Rightarrow$ mechanical instability of nuclear matter

$\Rightarrow$ phase transition to quark mater

Parameterization

- **Current quark mass (flavor blind for simplicity)**

$$m = \frac{m_u + m_d}{2} = 3.5 \ MeV$$

- **Chiral condensate in the vacuum**

$$\langle \bar{q}q \rangle = \frac{\partial \Omega}{\partial m} - \underbrace{\frac{\partial \Omega_{free}}{\partial m}}_{\textbf{regularization}} = -(250 \ MeV)^3$$

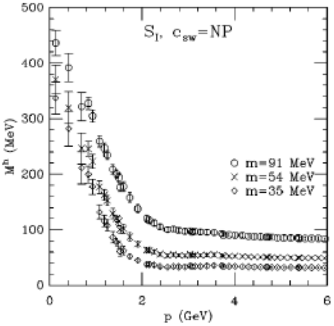
- **Momentum dependent mass in the vacuum**

$$m_{\mathbf{k}} = m + \sigma g_{\mathbf{k}}$$

$$m_{\mathbf{k}=0} = 400 \text{ MeV}$$

m, G_S, Λ – fixed

J. Skullerud, D. B. Leinweber, and A. G. Williams, Phys. Rev. D 64 , 074508 (2001)



- **Phase transition @ BEC of sexaquarks**

$$\mu_c = M_s/2 = 1027 \text{ MeV} \Rightarrow G_D - \text{fixed}$$

Solution of mean-field equations @ $T = 0$

- Destruction of CFL phase

$$M_{S\mathbf{k}} > M_c \equiv 2\sqrt{\Delta k_F}$$

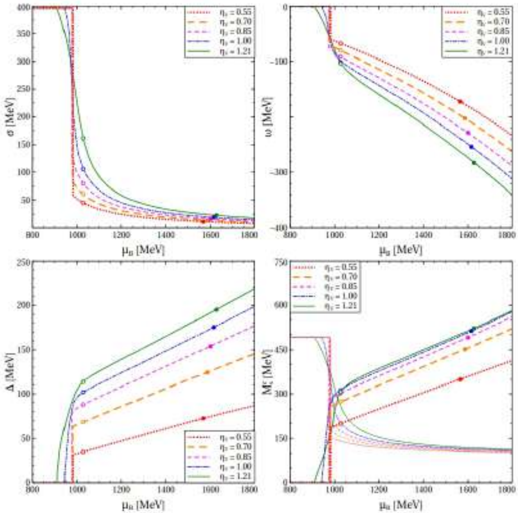
$$M_{S\mathbf{k}} = m_s + g_{\mathbf{k}}\sigma$$

T. Schafer, F. Wilczek, PRD 60, 074014 (1999)

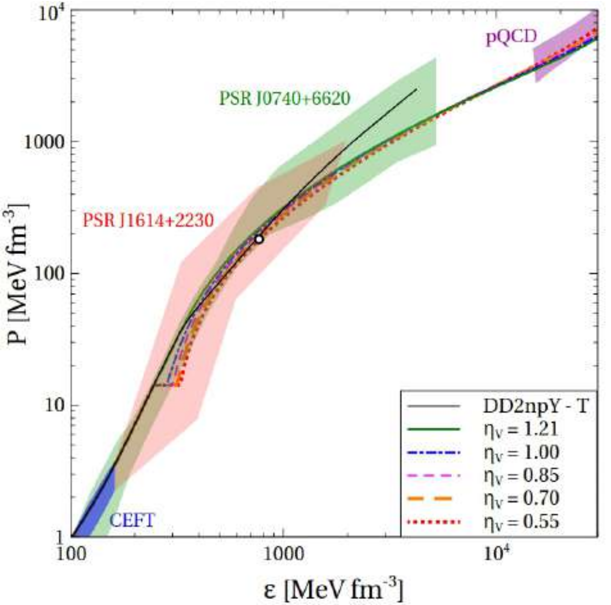
M. G. Alford, Jet al., Nucl. Phys. B 558, 219 (1999)

H. Abuki, Prog. Theor. Phys. 110, 937 (2003)

Does not happen for the chosen values of the model parameters

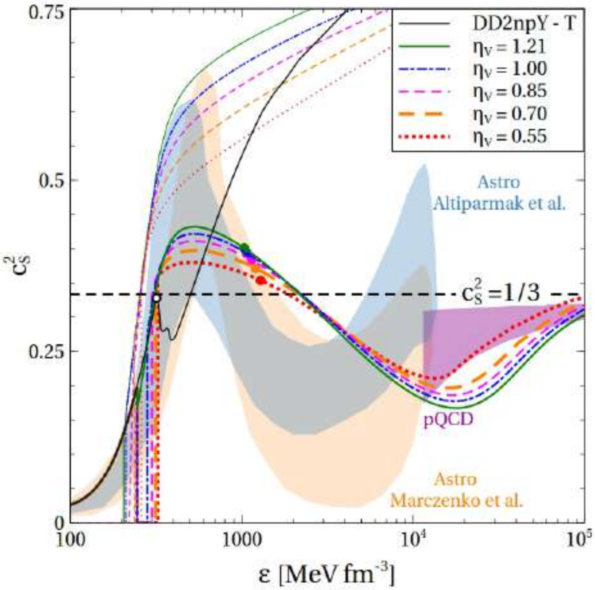


Pressure @ $T = 0$

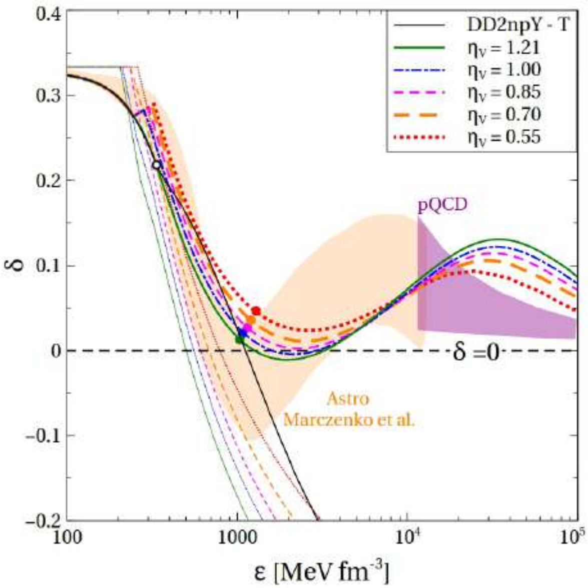


pQCD
PSR J0740+6620
PSR J1614+2230
CEFT
P [MeV fm^{-3}]
ε [MeV fm^{-3}]
DD2npY - T
$\eta_V = 1.21$
$\eta_V = 1.00$
$\eta_V = 0.85$
$\eta_V = 0.70$
$\eta_V = 0.55$

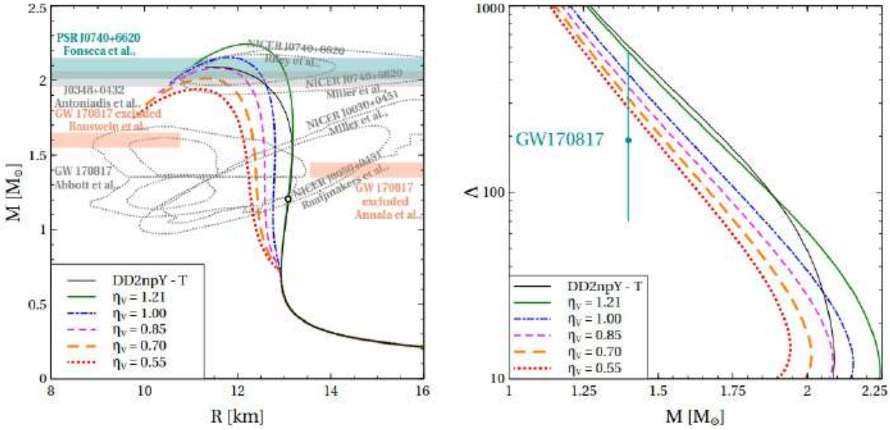
Speed of sound @ $T = 0$



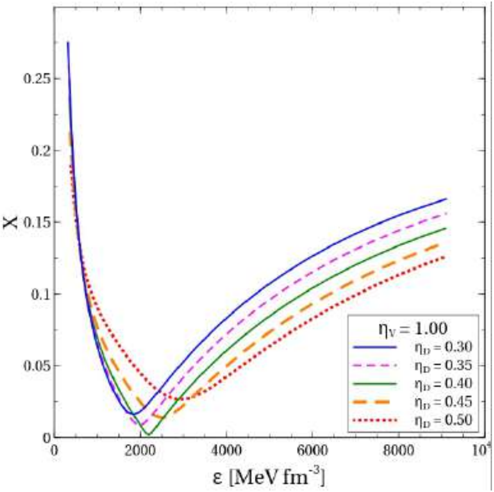
Dimensionless interaction measutre @ $T = 0$



Neutron stars with CFL cores

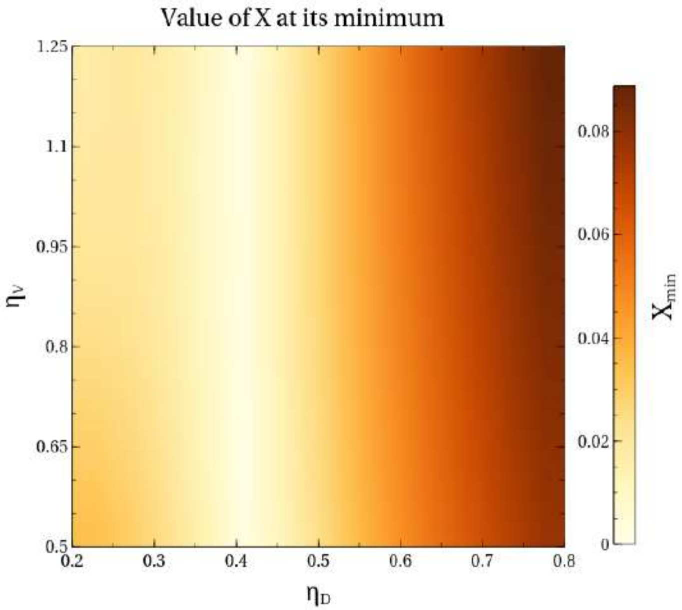


Is CFL quark matter nearly conformal?

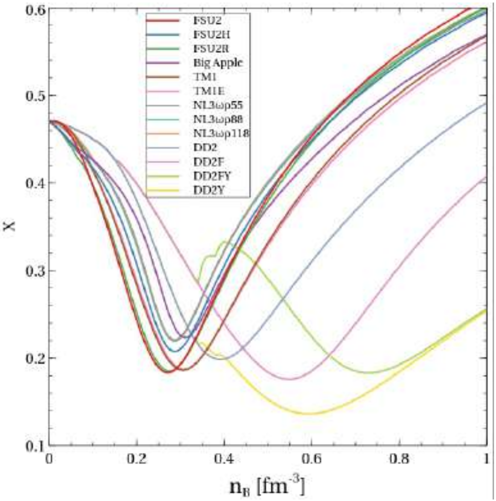


$$X \equiv \sqrt{\delta^2 + \left(c_S^2 - \frac{1}{3}\right)^2}$$

Is CFL quark matter nearly conformal?



Comparison to hadron matter



$$X_{\rm hadrons}^{\rm min} \simeq 0.15$$

Scale invariance of a fermioic degree of freedom

- Scale invariance and single particle energy

$$\begin{cases} T \to \lambda T \\ \mu \to \lambda \mu \\ \mathbf{k} \to \lambda \mathbf{k} \end{cases} \quad \Rightarrow \quad \Omega = -Td \int \frac{\mathbf{k}}{(2\pi)^3} \ln \left[1 + \exp\left(\frac{\mu - E}{T} \right) \right] \to \lambda^4 \Omega$$

$$\Downarrow$$

$$E \to \lambda E$$

- Single particle measure of scale invariance

$$\phi \equiv 1 - \frac{\partial \ln E}{\partial \ln |\mathbf{k}|}$$

$\phi = 0$ for a conformal degree of freedom

Microscopic criterion of approximate scale invariance

- Characteristic

$$\phi = const \quad \Rightarrow \quad E \propto |\mathbf{k}|^{1-\phi}$$

- Equation of state

$$n = d \int \frac{d\mathbf{k}}{(2\pi)^3} \propto k_F^3, \quad \varepsilon = d \int \frac{d\mathbf{k}}{(2\pi)^3} E \propto k_F^{4-\phi} \propto n^{(4-\phi)/3}$$

$$p = \mu n - \varepsilon = n \frac{d\varepsilon}{dn} - \varepsilon = \frac{1-\phi}{3} \varepsilon$$

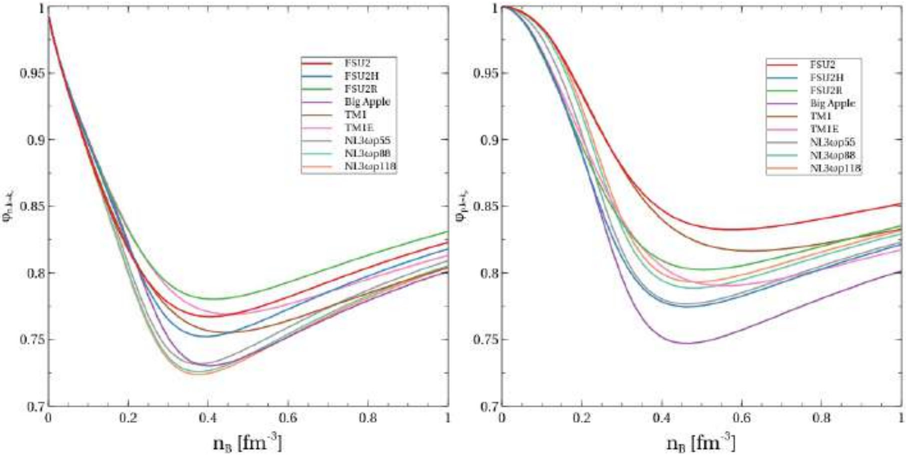
$$\delta = \frac{\phi}{3}, \quad c_S^2 = \frac{1-\phi}{3}$$

$$X = \frac{\sqrt{2}\phi}{3}$$

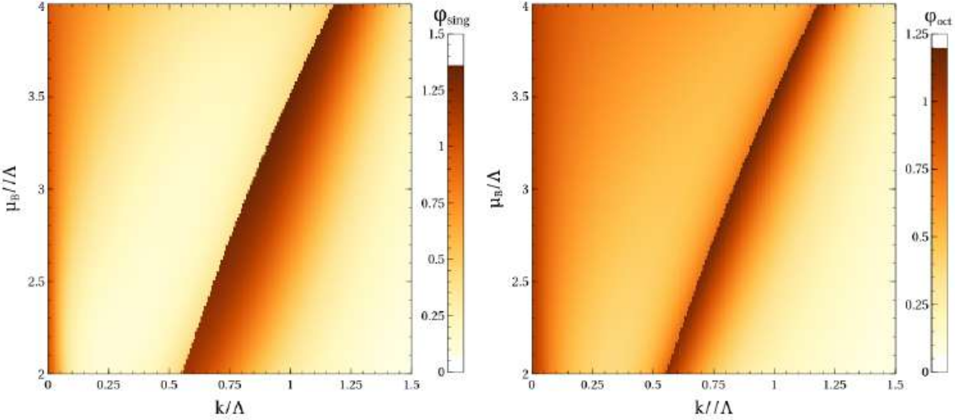
- Approximate scale invariance

$$X < X_{\mathrm{hadrons}}^{\mathrm{min}} \simeq 0.15 \quad \Rightarrow \quad \phi \lesssim \frac{1}{3}$$

Hadron matter is not scale invariant (as expected)

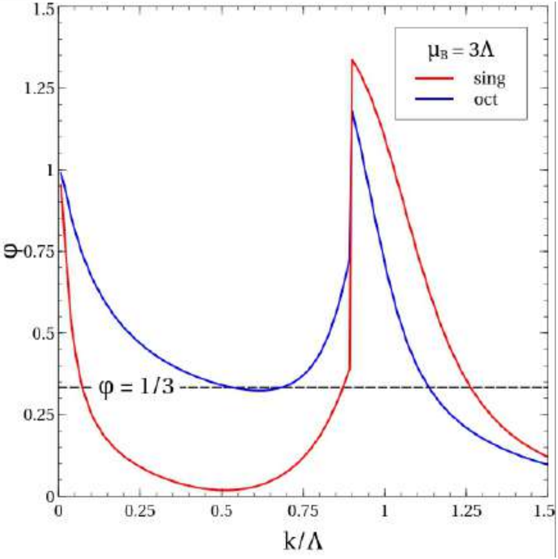


CFL quark matter as well



Pseudoconformal behavior

CFL quark matter as well



Pseudoconformal behavior

Conclusions

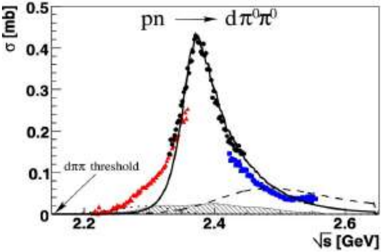
- Nonlocal NJL model for three-flavor quarks matter with vector repulsion and color-superconductivity
- Agreement with the observational data of neutron stars
- CFL quark matter is unlikely to be conformal but exhibits pseudoconformality

Conclusions

Thank you for your attention

Six-quark states

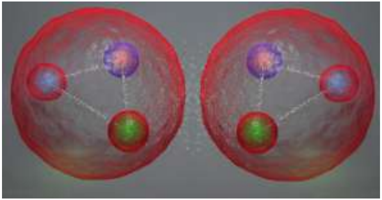
	$\mathbf{N}_f = 2$	$\mathbf{N}_f = 3$
state	d*(2380)	S
content	uuuddd	uuddss
status	observed (WASA-at-COS Collaboration)	being searched



P. Adlarson, et al., Phys. Rev. Lett., 106 (2011)

Double strange six-quark state

- **Dibaryon molecule of two Λs**
  1. Weakly bound or resonance nature
  2. Large mass
  3. Irrelevant for the phenomenology of dense matter



- **Multiquark state (sexaquark)**
  1. Deeply bound
  2. Not too large or small mass
  3. Important for the phenomenology of dense matter

- **Stability with respect to strong processes**

$$M_S < 2M_\Lambda = 2230 \text{ MeV} \Rightarrow \text{no } S \to \Lambda + \Lambda \text{ decay}$$

- **Stability with respect to weak processes**

$$M_S < M_\Lambda + M_N = 2054 \text{ MeV} \Rightarrow \text{no } S \to \Lambda + N + l \text{ decay}$$

Sexaquark: what to expect?

- **Electrically neutral color, flavor, spin singlet**

  completely antisymmetric wave function ψ_S $\Rightarrow$ compact deeply bound state

- **Chromomagnetic and chromoelectric contributions from ψ_S**

  $M_S = 1883$ MeV $\Rightarrow$ only the double weak decay $S \to 2N + 2l$ is allowed

  F. Buccella, PoS CORFU2019, 024 (2020)

Weakly-interacting state with lifetime of the Universe?
Dark matter candidate within QCD?

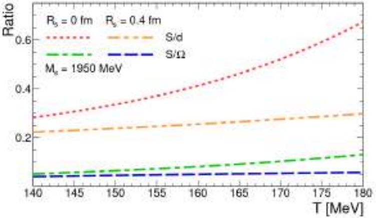
G. R. Farrar, J.Theor.Phys. 42 (2003) 1211-1218

- **Sexaquark in the Early Universe QCD transition**

  thermal production at $T = 156.6$ MeV

  $\Downarrow$

  $\varepsilon_S/\varepsilon_{tot}$ compatible to the baryons-to-dark matter ratio

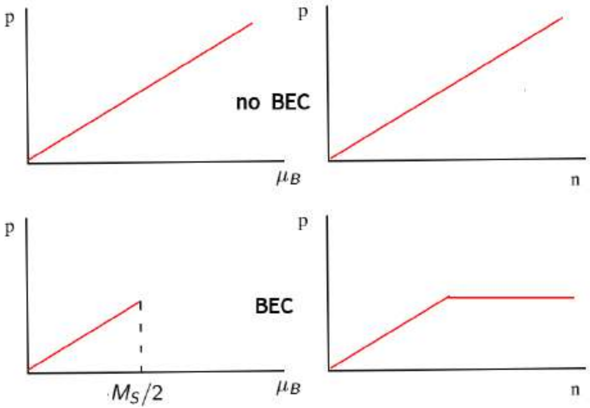


D. Blaschke et al., Journal of Modern Physics A, Vol. 36, No. 25, 2141005 (2021)

Sexaquarks condensation in nuclear matter

no BEC

BEC



BEC of sexaquarks $\Rightarrow$ mechanical instability of nuclear matter

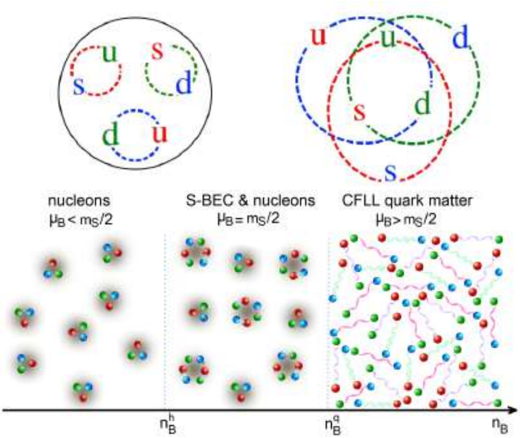
$\Rightarrow$ phase transition to quark mater

Sexaquarks and CFL quark matter

- **Diquarks as color antitriplets**

$3 \otimes 3 = \bar{3} \oplus 6 \quad \Rightarrow$ **3 diquarks interact** *as 3 quarks* $\Rightarrow$ **dissociation if S triggers liberation of diquarks = CFL quark matter**

- **Micro**
- **Macro**



Onset of quark matter

- **Weak decays stability**

  $$M_S < M_N + M_\Lambda \quad \Rightarrow \quad M_{\rm onset} < 0.7\ M_\odot$$

- **Strong decays stability**

  $$M_S < 2M_\Lambda \quad \Rightarrow \quad M_{\rm onset} < 1.31\ M_\odot$$

- **Instability**

  no sexaquark onset